MATHEMATICS (STANDARD)

SOLUTIONS
SAMPLE QUESTION PAPER 6

CBSE Class X Examination

Section 'A'

1. (*c*) **2.** (*a*) **3.** (*b*) **4.** (*a*)
5. (*d*) **6.** (*d*) **7.** (*d*) **8.** (*b*)
9. (*d*) **10.** (*b*)

Fill in the blanks

11. $k = 8$

Explanation

Given equation is $x^2 - 3x + k - 10 = 0$

Here $a = 1, b = -3$ and $c = k - 10$

Product of roots $= \frac{c}{a}$

$$-2 = \frac{k-10}{1}$$

$$\Rightarrow \quad k = 8$$

12. $k = -11$

Explanation

Putting $x = 3$ in $3x^2 (k - 1)x + 9 = 0$

$3(3)^2 + (k - 1)(3) + 9 = 0$

$$27 + 3k - 3 + 9 = 0$$

$$3k = -33$$

$$k = -11$$

13. $x = -2$

Explanation

Given equation is $3^{x+2} + 3^{-x} = 10$

$$\Rightarrow \quad 3^x . 3^{+2} + 3^{-x} = 10$$

$$\Rightarrow \quad 3^x \times 9 + \frac{1}{3^x} = 10$$

Let $3^x = y$

$$9y + \frac{1}{y} = 10$$

$$\Rightarrow \quad 9y^2 - 10y + 1 = 0$$

$$\Rightarrow \quad 9y^2 - 9y - y + 1 = 0$$

$$9y(y - 1) - 1(y - 1) = 0$$

$$(y - 1)(9y - 1) = 0$$

$$y = 1, \frac{1}{9}$$

$$\therefore \quad 3^x = 1$$

$$\Rightarrow \quad x = 0$$

or $$3^x = \frac{1}{9} = 3^{-2}$$

$$x = -2$$

14. $c = 6$

Explanation

$x^2 - 5x + c = 0$

Here $a = 1, b = -5$ and $c = c$

$$\therefore \quad \alpha + \beta = \frac{-b}{a} = 5$$

Adding
$$\begin{array}{c} \alpha + \beta = 5 \\ \underline{\alpha - \beta = 1} \\ 2\alpha = 6 \end{array} \quad \text{or} \quad \alpha = 3, \beta = 2$$

Now product of roots $\alpha\beta = \frac{c}{a}$

$$3 \times 2 = \frac{c}{1}$$

$$\therefore \quad c = 6$$

15. $k = 6$

Explanation

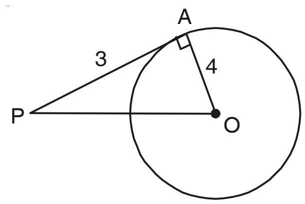
For infinitely many solutions

$$\frac{2}{4} = \frac{3}{k} = \frac{5}{10}$$

$$\Rightarrow \quad k = 6$$

Answer the following.

16. Applying pythagoras theorem.



$PO^2 = 3^2 + 4^2$

$= 25$

$\therefore \quad PQ = 5$ units

17. Cos (40 + θ) – sin (50 – θ)

= cos [90 – (50 – θ)] – sin (50 – θ)

= sin (50 – θ) – sin (50 – θ)

= 0

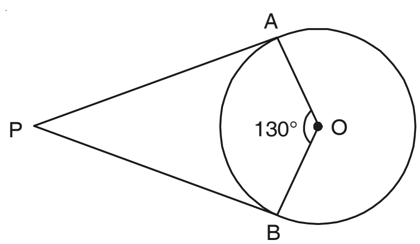
18. sin 43° cos (90° – 43°) + cos 43° sin (90° – 43°)

= sin 43° . sin 43 + cos 43° cos 43°

$= \sin^2 43 + \cos^2 43$

= 1

19.



Given $\angle AOB = 130$

Also $\angle APB + \angle AOB = 180°$

$\therefore \quad \angle APB = 180° - 130° = 50°$

20. $\sec\theta + \tan\theta = x$ ... (i)

Rationalising

$$\frac{(\sec\theta+\tan\theta)}{1}\times\frac{(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta)} = x$$

$$\Rightarrow \frac{\sec^2\theta-\tan^2\theta}{\sec\theta-\tan\theta} = \frac{x}{1}$$

$(\sec^2\theta - \tan^2 x = 1)$

$$\Rightarrow \sec\theta - \tan\theta = \frac{1}{x} \quad \text{... (ii)}$$

from (i) and (ii)

sec θ + tan θ = x

$$\sec\theta - \tan\theta = \frac{1}{x}$$

$$\text{Adding } 2\sec\theta = x + \frac{1}{x}$$

$$2\sec\theta = \frac{x^2+1}{x}$$

$$\sec\theta = \frac{x^2+1}{2x}$$

Section 'B'

21. LCM $(p, q) = a^3b^3$ **[½]**

HCF $(p, q) = a^2b$ **[½]**

LCM $(p, q) \times$ HCF $(p, q) = a^5b^4$

$= (a^2b^3)(a^3 b) = pq$ **[1]**

22. ***(i)*** Let the cost of one book be ₹ x and that of one pen be ₹ y. Then,

Cost of 5 books + cost of 7 pen = ₹ 79

$\Rightarrow \quad 5x + 7y = 79$ ...(*i*)

Cost of 7 books + Cost of 5 pens = ₹ 77

$\Rightarrow \quad 7x + 5y = 77$ ...(*ii*)

Adding (*i*) and (*ii*),

$5x + 7y = 79$

$+ 7x + 5y = 77$

$12x + 12y = 156$

$12(x + y) = 156$

$$x + y = \frac{157}{12} = 13$$ **[½]**

$100(x + y) = 13 \times 100 = 1300$ ₹

∴ 100 books and 100 pen will cost 1300 ₹. **[1]**

(ii) Pair of linear equation in two variables. **[½]**

23. First 8 multiples of 3

3, 6, 9, 12, 15, 18, 21, 24 **[1]**

$S = 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24$

Thses numbers are in A.P.

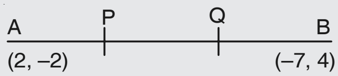
where $a = 3$, $d = 3$ and $n = 8$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_8 = \frac{8}{2}\left[2\times3+(8-1)3\right]$$

$$S_8 = 4\,[6 + 21]$$
$$S_8 = 4 \times 27 = 108$$

∴ Thus, sum of first 8 multiples of 3 of 108. **[1]**

24. 

P divides AB 1 : 2

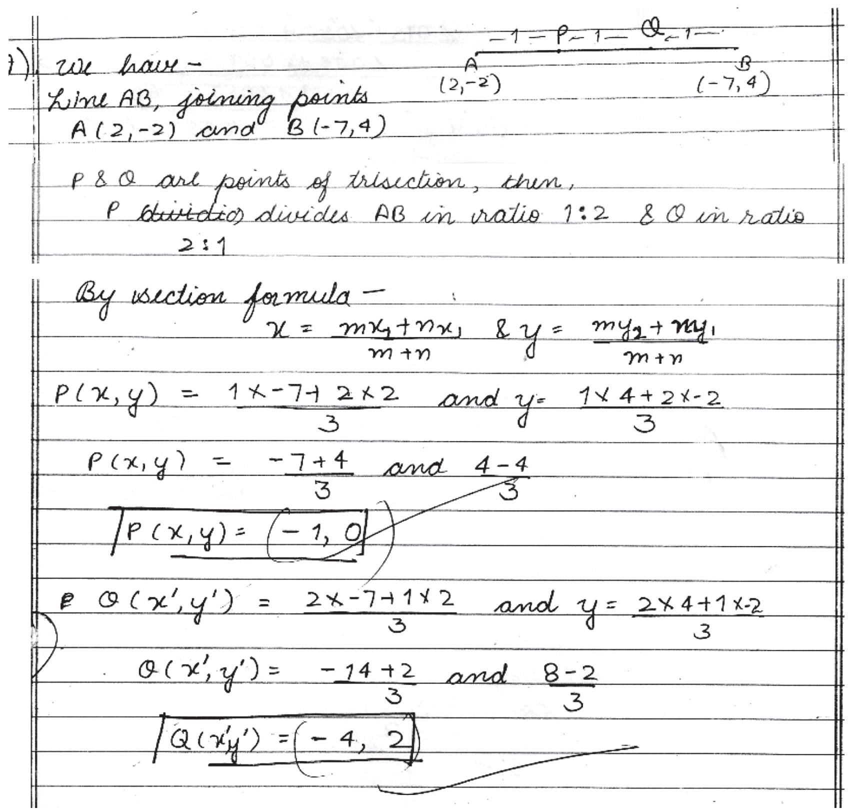
∴ Coordinates of P are : (–1, 0) **[1]**

Q is mid-point of PB

∴ Coordinates of Q are : (–4, 2) **[1]**

[CBSE Marking Scheme, 2016]

CBSE Topper's Solution, 2016



We have –
Line AB, joining points A (2, –2) and B (–7, 4)

P & Q are points of trisection, then, P divides AB in ratio 1 : 2 & Q in ratio 2 : 1

By section formula –

$$x = \frac{mx_2 + nx_1}{m+n} \quad \& \quad y = \frac{my_2 + ny_1}{m+n}$$

$$P(x, y) = \frac{1 \times -7 + 2 \times 2}{3} \text{ and } y = \frac{1 \times 4 + 2 \times -2}{3}$$

$$P(x, y) = \frac{-7+4}{3} \text{ and } \frac{4-4}{3}$$

$$P(x, y) = (-1, 0)$$

$$Q(x', y') = \frac{2 \times -7 + 1 \times 2}{3} \text{ and } y = \frac{2 \times 4 + 1 \times -2}{3}$$

$$Q(x', y') = \frac{-14+2}{3} \text{ and } \frac{8-2}{3}$$

$$Q(x', y') = (-4, 2)$$

25. Total number of all possible outcomes = 20

Numbers multiple of 3 are : 3, 6, 9, 12, 15, 18

Numbers multiple of 7 are : 7, 14 **[1]**

Numbers multiple of 3 of 7 are : 3, 6, 7, 9, 12, 14, 15, 18

Let A be the event that the number on the drawn ticket is a multiple of 3 or 7.

Then, the number of favourable outcomes = 8

∴ P (that the drawn ticket is a multiple of 3 or 7) = P (A) = $\frac{8}{20} = \frac{2}{5}$ **[1]**

26. Possibilities are HH, HT, TH, TT **[1]**

$$P\text{ (HH or TT)} = \frac{2}{4} = \frac{1}{2}$$ **[1]**

Section 'C'

27. Let us consider two numbers 225 and 60

We obtain 225 = 60 × 3 + 45 **[1]**

[∵ 225 > 62]

60 = 45 × 1 + 15

[∵ 60 > 45]

$$45 = 15 \times 3 + 0$$

$[\because \quad 45 > 15]$

Thus, HCF (225, 60) $= 15$ **[1]**

Also, $225 \times 5 - 10x = 15$

$\Rightarrow \quad 10x = 1125 - 15$

$\Rightarrow \quad x = \dfrac{1110}{10} = 111$ **[1]**

28. Since two zeroes are $3\sqrt{2}$ and $-3\sqrt{2}$, therefore $\left(x-3\sqrt{2}\right)\left(x+3\sqrt{2}\right) = x^2 - 18$ is a factor of the given polynomial.

Now, we divide the given polynomial by $x^2 - 18$.

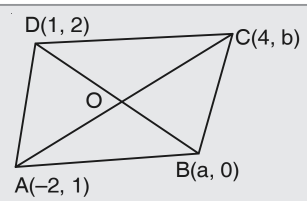
So, $4x^4 + x^3 - 72x^2 - 18x = (x^2 - 18)(4x^2 + x)$

$= (x^2 - 18)[x(4x + 1)]$ **[1]**

$= \left(x-3\sqrt{2}\right)\left(x+3\sqrt{2}\right)x(4x+1)$ **[1]**

The other zeroes of the given polynomial are 0 and $-\dfrac{1}{4}$. **[1]**

29.



ABCD is a parallelogram

$\therefore$ diagonals AC and BD bisect each other

Therefore

Mid point of BD is same as mid point of AC

$\Rightarrow \quad \left(\dfrac{a+1}{2}, \dfrac{2}{2}\right) = \left(\dfrac{-2+4}{2}, \dfrac{b+1}{2}\right)$

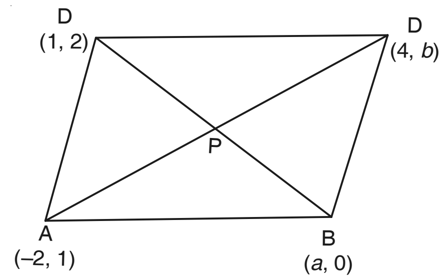
$\Rightarrow \quad \dfrac{a+1}{2} = 1$ and $\dfrac{b+1}{2} = 1$

$\Rightarrow \quad a = 1, b = 1$. Therefore length of sides are $\sqrt{10}$ units each.

[CBSE Marking Scheme, 2016]

Expert's Solution:

We know that diagonals of parallelogram bisect each other.



$\therefore$ Midpoint of diagonal AC

$$\left(\frac{-2+4}{2}, \frac{1+b}{2}\right) = \left(1, \frac{1+b}{2}\right)$$ **[½]**

Mid-point of diagonal BD

$$\left(\frac{a+1}{2}, \frac{0+2}{2}\right) = \left(\frac{a+1}{2}, 1\right)$$ **[½]**

Mid point of diagonal AC $\cong$ mid point of diagonal BD

$1 = \dfrac{a+1}{2}$ and $\dfrac{1+b}{2} = 1$

$2 = a + 1$ and $1 + b = 2$

$\therefore \quad a = 1$ and $b = 1$

$$AB = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

$$= \left|\sqrt{(1+2)^2 + (0-1)^2}\right|$$

$$= \left|\sqrt{9+1}\right| = \sqrt{10} \text{ unit}$$

$$BC = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_{1l})^2}\right|$$

$$= \left|\sqrt{(4-1)^2 + (1-0)^2}\right|$$

$$= \left|\sqrt{9+1}\right| = \sqrt{10} \text{ unit}$$

ABCD is a parallelogram (Given)

$AB = CD = \sqrt{10}$ unit **[½]**

$BC = AD = \sqrt{10}$ unit **[½]**

OR

Given, $\quad AP = \dfrac{3}{7} AB$

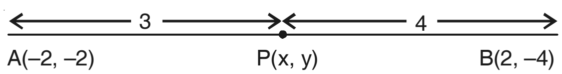
$$\Rightarrow \quad AP = \frac{3}{7}(AB + BP)$$

$$\Rightarrow \quad 7\,AP = 3\,AP + 3\,BP \quad \textbf{[1]}$$

$$\Rightarrow \quad 4\,AP = 3\,BP$$

$$\Rightarrow \quad \frac{AP}{BP} = \frac{3}{4}$$

$$\Rightarrow \quad AP : BP = 3 : 4 \quad \textbf{[1]}$$



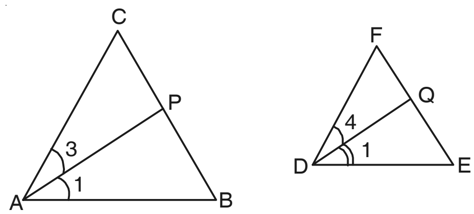
Now, coordinates of P

$$= \left(\frac{3\times 2+4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right)$$

[by internal section formula]

$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right) \quad \textbf{[1]}$$

30.



(*i*) $\Delta ABC \sim \Delta DEF$

$\therefore$ $\angle A = \angle D$ (corresponding angles)

$2\angle 1 = 2\angle 2$

or, $\angle 1 = \angle 2$

also $\angle B = \angle E$ (corresponding angles)

$\Delta APB \sim \Delta DQE$ **[1]** (by AA similarity)

or, $\dfrac{AP}{DQ} = \dfrac{AB}{DE}$ **[1]**

(*ii*) $\because$ $\Delta ABC \sim \Delta DEF$

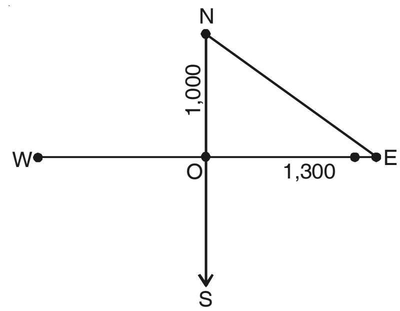
$\therefore$ $\angle A = \angle D$

and $\angle C = \angle F$

or, $2\angle 3 = 2\angle 4$ or, $\angle 3 = \angle 4$

$\therefore$ $\Delta CAP \sim \Delta FDQ$ **[1]** (By AA similarity)

OR



[1]

Distance covered by first aeroplane due North after two hours = 500 × 2 = 1,000 km. **[½]**

Distance covered by second aeroplane due East after two hours = 650 × 2 = 1,300 km. **[½]**

Distance between two aeroplane after 2 hours

$$NE = \sqrt{ON^2 + OE^2}$$

$$= \sqrt{(1000)^2 + (1300)^2}$$

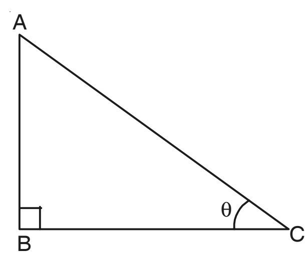
$$= \sqrt{1000000 + 1690000}$$

$$= \sqrt{2690000}$$

$$= 1640.12 \text{ km}$$

31. Expert's solution :

Given, 4 than $\theta = 3$



Find the value of $= \dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}$

Given, $4\tan\theta = 3$

$$\tan\theta = \frac{3}{4} \quad \textbf{[1]}$$

$$\frac{AB}{BC} = \frac{3}{4}$$

$\therefore$ AB = 3k and BC = $4k$

In ΔABC, $\angle B = 90^\circ$

$\therefore$ $AC^2 = AB^2 + BC^2$

(By Pythagorus theorem)

$$AC^2 = (3k)^3 + (4k)^2$$
$$AC^2 = 9k^2 + 16k^2 = 25k^2$$
$$AC = 5k$$

$$\sin\theta = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$ **[½]**

$$\cos\theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$ **[½]**

$$\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4\times\frac{3}{5} - \frac{4}{5} + 1}{4\times\frac{3}{5} + \frac{4}{5} - 1}$$

$$= \frac{\frac{12}{5} - \frac{4}{5} + 1}{\frac{12}{5} + \frac{4}{5} - 1}$$

$$= \frac{12 - 4 + 5}{12 + 4 - 5} = \frac{13}{11}$$

$$\therefore \quad \frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{13}{11}$$ **[1]**

OR

$$\text{LHS} = \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$

$$= \frac{\cos A}{1 - \left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1 - \left(\frac{\cos A}{\sin A}\right)}$$ **[1]**

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$ **[1]**

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$= \cos A + \sin A$$

$$= \sin A + \cos A = \text{RHS}$$ **[1]**

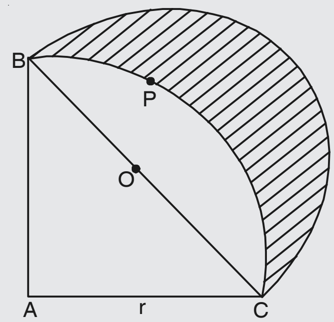
32. We know, $AC = r$

In ΔACB, $BC^2 = AC^2 + AB^2$

$\Rightarrow \quad BC = AC\sqrt{2}$

$(\because AB = AC)$

$\Rightarrow \quad BC = r\sqrt{2}$ **[1]**



Required area = *ar* (ΔACB) + *ar* (semicircle on BC as diameter) – *ar* (quadrant ABPC)

$$= \frac{1}{2}\times r\times r + \frac{1}{2}\times\pi\times\left(\frac{r\sqrt{2}}{2}\right)^2 - \frac{1}{4}\pi r^2$$ **[1]**

$$= \frac{r^2}{2} + \frac{\pi r^2}{4} - \frac{\pi r^2}{4}$$

$$= \frac{r^2}{2} = \frac{196}{2}\text{cm}^2 = 98 \text{ cm}^2$$ **[1]**

[CBSE Marking Scheme, 2018]

33. Volume of metal in 504 cones

$$= 504\times\frac{1}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times 3 \text{ cm}$$

$$\therefore \text{Value of sphere} = \frac{4}{3}\times\frac{22}{7}\times r^3$$ **[1]**

$$= 504\times\frac{1}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times 3$$

$$= \frac{4}{3}\times\frac{22}{7}\times r^3$$ **[1]**

$r = 10.5$ cm. $\quad\therefore\quad$ diameter = 21 cm

$$\text{Surface area} = 4\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}$$

$$= 1386 \text{ cm}^2$$ **[1]**

[CBSE Marking Scheme, 2015]

CBSE Topper's Solution, 2015

(17) Diameter of a cone = 3.5 cm

⇒ radius(R) = 3.5/2 cm

Height of cone = H = 3 cm

Here, 504 cones are melted to form a sphere.

Let the radius of sphere = R

Now, Vol^m of sphere = Vol^m of 504 cones

or, $\frac{4}{3}\pi R^3 = 504 \times \frac{1}{3} \times \pi \times 3 \times \frac{3.5}{2} \times \frac{3.5}{2}$

or, $R^3 = \frac{126 \times 3 \times 35 \times 35}{4 \times 100} = \frac{7 \times 7 \times 7 \times 18 \times 3}{16}$

or, $R = \sqrt[3]{\frac{7\times7\times7\times3\times3\times3}{2\times2\times2}} = \frac{7\times3}{2} = \frac{21}{2}$ cm

∴ Diameter = 2R = $2 \times \frac{21}{2}$ = 21 cm

S.A of hemisphere = $4\pi R^2$

$= 4\pi \times \frac{21}{2} \times \frac{21}{2} = \frac{22}{7} \times 21 \times 21$

$= 66 \times 21$

$= 1386$ cm²

OR

Let the area that can be irrigated in 30 minute be A m^2.

Water flowing in canal in 30 minutes

$$= \left(10,000 \times \frac{1}{2}\right) \text{m} = 5000 \text{ m}$$

Volume of water flowing out in 30 minutes = $(5000 \times 6 \times 1.5)\text{m}^3 = 45000 \text{ m}^3$ ...(*i*) **[1]**

Volume of water required to irrigate the field

$$= A \times \frac{8}{100} \text{m}^3 \quad ...(ii)$$ **[1]**

Equating (*i*) and (*ii*), we get

$$A \times \frac{8}{100} = 45000$$

$$A = 562500 \text{ m}^2.$$ **[1]**

[CBSE Marking Scheme, 2018]

34.

Salary (in thousand ₹)	No. of persons (f)	cf
5–10	49	49
10–15	133	182
15–20	63	245
20–25	15	260
25–30	6	266
30–35	7	273
35–40	4	277
40–45	2	279
45–50	1	280

[2]

$$\frac{N}{2} = \frac{280}{2} = 140$$

Median class is 10–15

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - C\right)$$

$$= 10 + \frac{5}{133}(140 - 49) = 10 + \frac{5 \times 91}{133} = 13.42$$

Median salary is ₹ 13.42 thousand or ₹ 13420 (approx) **[1]**

[CBSE Marking Scheme, 2016]

Section 'D'

35. $x + 2y = 5$ or, $y = \frac{5 - x}{2}$

x	1	3	5
y	2	1	0

[1]

$2x - 3y = -4$

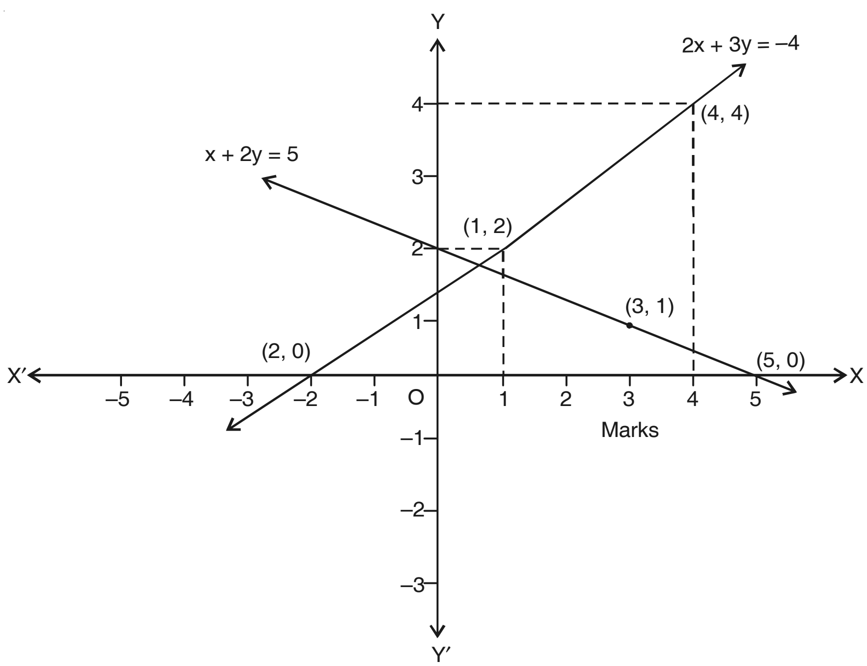
Or, $y = \frac{2x + 4}{3}$]

x	1	4	–2
y	2	4	0

[1]

lines meet x-axis at (5, 0) and (–2, 0) respectively.

[CBSE Marking Scheme, 2015]



OR

Let the time taken by the tap to fill the tank of smaller diameter = x hours

∴ Time taken by the tap of larger diameter to fill the tank = $(x - 9)$ hours

∴ Work done by the tap of smaller diameter in one hour = $\frac{1}{x}$ **[½]**

and the work done by the tap of larger diameter in one hour = $\frac{1}{x - 9}$ **[½]**

Thus, the work done by the two taps together in 1 hour

$$= \frac{1}{x} + \frac{1}{x - 9}$$

$= \frac{x-9+x}{x(x-9)} = \frac{2x-9}{x(x-9)}$ **[1]**

The two tap together can fill the tank in $\frac{x(x-9)}{2x-9}$ hours.

According to the information,

$$\frac{x(x-9)}{2x-9} = 6 \text{ hours (given)}$$

$\Rightarrow \quad x^2 - 9x = 12x - 54$

$\Rightarrow \quad x^2 - 21x + 54 = 0$ **[1]**

$\Rightarrow \quad x^2 - 3x - 18x + 54 = 0$

$\Rightarrow x(x-3) - 18(x-3) = 0$

$\Rightarrow \quad (x-3)(x-18) = 0$

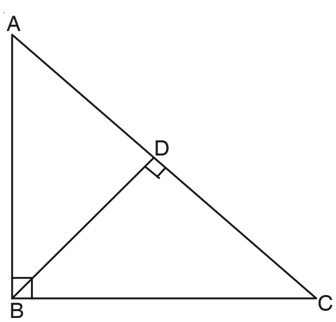
$\Rightarrow$ Either $x - 3 = 0$

or $x - 18 = 0$

$\Rightarrow$ Either $x = 3$ or $x = 18$ **[1]**

When x = 3, then the tap of smaller diameter can fill the tank in 3 hours and the tap of the larger diameter can fill the tank in 3 – 9 = –6, which is rejected as time to fill the tank cannot be negative.

When x = 18, then the tap of smaller diameter can fill the tank in 18 hours and the tap of the larger diameter can fill the tank in 18 – 9 = 9 hours.

36. 

Given : In triangle ABC,

$\angle ABC = 90^\circ$

To Prove : $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : In ΔABD and ΔABC,

$\angle ABC = \angle ADB = 90^\circ$ (By construction)

$\angle BAC = \angle BAD$ (Common angle)

$\therefore \quad \Delta ABD \sim \Delta ACB$ **[1]** (AA Similarity)

$\therefore \quad \frac{AB}{AC} = \frac{BD}{BC} = \frac{AD}{AB}$

$\frac{AB}{AC} = \frac{AD}{AB}$

$AB^2 = AC \times AD \quad ...(i)$

[1]

In ΔABC and ΔBCD

$\angle ACB = \angle BCD$ (Common angle)

$\angle ABC = \angle BDC = 90^\circ$ (By construction)

$\therefore \quad \Delta ABC \sim \Delta BDC$ (AA Similarity)

$\frac{AB}{BD} = \frac{BC}{DC} = \frac{AC}{BC}$

$\frac{BC}{DC} = \frac{AC}{BC}$

$BC^2 = AC \times DC \quad ...(ii)$ **[1]**

Adding equation (*i*) and (*ii*),

$AB^2 + BC^2 = AC \times AD + AC \times DC$

$AB^2 + BC^2 = AC(AD + DC)$

$AB^2 + BC^2 = AC \times AC$

$AB^2 + BC^2 = AC^2$ **[1]**

OR

Given : A ΔABC, in which AD is the median.

To prove : $AB^2 + AC^2 = 2(AD^2 + BD^2)$

Construction : Draw $AE \perp BC$.

Proof : In right ΔAEB, we have

$AB^2 = AE^2 + BE^2 \quad ...(i)$

[By Pythagoras Theorem]

In right ΔAEC, we have

$AC^2 = AE^2 + CE^2 \quad ...(ii)$

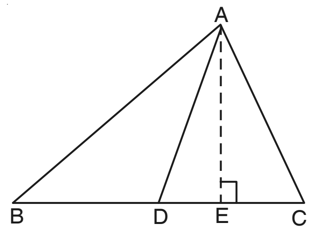
[By Pythagoras Theorem]

Adding (*i*) and (*ii*), we get

$AB^2 + AC^2 = (AE^2 + BE^2) + (AE^2 + CE^2)$

$\Rightarrow \quad AB^2 + AC^2 = 2AE^2 + (BE^2 + CE^2) \quad ...(iii)$ **[1]**

In right ΔAED, we have



[1]

$$AD^2 = AE^2 + ED^2$$

[By Pythagoras Theorem]

$$\Rightarrow \quad AE^2 = AD^2 - ED^2 \quad ...(iv)$$

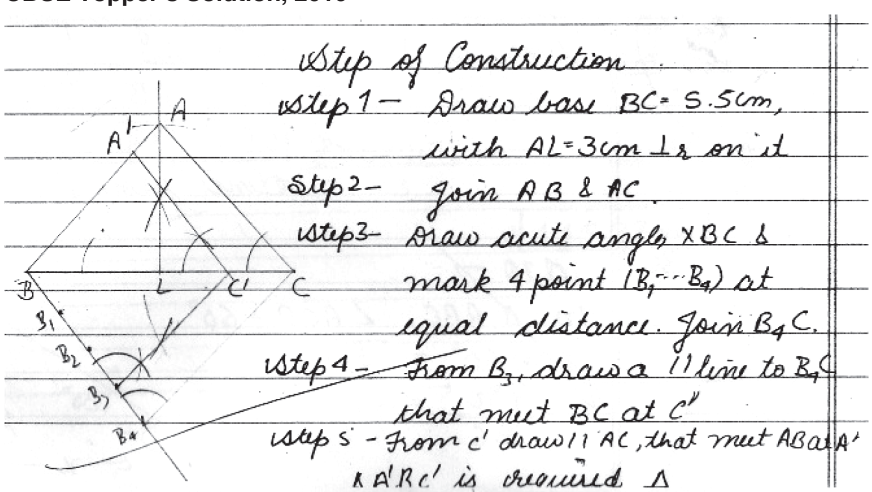
From (*iii*) and (*iv*), we get

$$AB^2 + AC^2 = 2(AD^2 - ED^2) + (BE^2 + CE^2)$$
$$= 2AD^2 - 2ED^2 + BE^2 + CE^2$$
$$= 2AD^2 + (BE^2 - ED^2) + (CE^2 - ED^2)$$
$$= 2AD^2 + (BE + ED).$$
$$(BE - ED) + (CE + ED)(CE - ED)$$
$$= 2AD^2 + (BE + ED).BD$$
$$+ DE.(CE - ED)$$
$$= 2AD^2 + BE + ED).BD$$
$$+ BD.(CE - ED) \quad \textbf{[1]}$$
$$[\because BD = CD]$$
$$= 2AD^2 + BD.(BE + ED + CE - ED)$$
$$= 2AD^2 + BD.(BE + CE)$$
$$= 2AD^2 + BD.BC$$
$$= 2AD^2 + BD(2BD)$$
$$[\because \text{AD is median}]$$
$$= 2(AD^2 + BD^2) \quad \textbf{[1]}$$

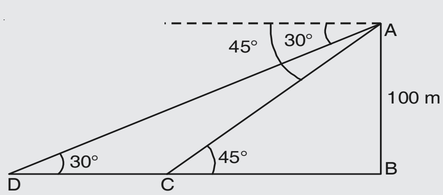
37. Correct Construction. **[4]**

[CBSE Marking Scheme, 2016]

CBSE Topper's Solution, 2016


Step of Construction
Step 1 – Draw base BC = 5.5cm, with AL=3cm ⊥r on it
Step 2 – Join AB & AC.
Step 3 – Draw acute angle XBC & mark 4 point (B1...B4) at equal distance. Join B4C.
Step 4 – From B3, draw a || line to B4C that meet BC at C'
Step 5 – From c' draw || AC, that meet AB at A'
ΔA'BC' is required Δ

38.



[1]

Let AB be the tower and ships are at points C and D.

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{AB}{BC} = 1$$

$$\Rightarrow \quad AB = BC \quad \textbf{[1]}$$

$$\text{Also, } \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{AB + CD} \quad \textbf{[1]}$$

$$\Rightarrow \quad AB + CD = \sqrt{3}AB$$

$$\Rightarrow \quad CD = AB\left(\sqrt{3} - 1\right)$$
$$= 100 \times (1.732 - 1)$$
$$= 73.2 \text{ m}$$

[CBSE Marking Scheme, 2018]

39. Slant height conical part

$= \sqrt{(2.8)^2 + (2.1)^2} = 3.5$ m **[1]**

Area of canvas/tent

$= 2\times\frac{22}{7}\times 2.8\times 3.5 + \frac{22}{7}\times 2.8\times 3.5\ \text{m}^2$

$= 92.4\ \text{m}^2$ **[1]**

Cost of 1500 tents $= 1500 \times 92.4 \times 120$

$=$ ₹ 16632000

Share of each school $= \frac{1}{50}\times 1663200$ ∘

$=$ ₹ 332640/- **[1]**

"Helping the needy" **[1]**

[CBSE Marking Scheme, 2016]

CBSE Topper's Solution, 2016

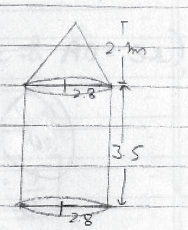
Section D

21) We have-

Radius of cylindrical and conical base = 2.8m.

Height of cylinder = 3.5m.

Height of conical part = 2.1 m



Slant height of conical part $= \sqrt{h^2+r^2} \Rightarrow \sqrt{(2.8)^2+(2.1)^2} \Rightarrow \sqrt{7.84+4.41}$

$l \Rightarrow \sqrt{12.25m^2} \Rightarrow l = 3.5m$

Total canvas required to make 1 tent = C.S.A of cylinder + C.S.A of cone

$\Rightarrow 2\pi rh + \pi rl$

$\Rightarrow \pi r(2h + l) \Rightarrow \frac{22 \times 2.8}{7}(2\times 3.5+3.5)\,m^2$

$\Rightarrow (22\times .4\times(7+3.5))\,m^2 \Rightarrow (8.8\times 10.5)\,m^2$

$\Rightarrow 92.4\ m^2$

Canvas required to make 1500 such tent

$= 92.4 \times 1500$

$138600.0\,m^2 = 138600\,m^2$

Total cost of making tent @ ₹120/m²

$\Rightarrow$ ₹120×138600

Amount shared by each school, when there are 50 school

$\frac{120 \times 138600}{50}$

₹12×27720

= ₹332640

Value- "Help the needy"

40.

C.I.	*f*	*c.f.*
0–10	5	5
10–20	x	$5 + x$
20–30	6	$11 + x$
30–40	y	$11 + x + y$
40–50	6	$17 + x + y$
50–60	5	$22 + x + y$

[2]

Here from table, $N = 22 + x + y = 40$

$\Rightarrow \quad x + y = 18 \quad ...(i)$

Since, Median = 31, which lies between 30–40

$\therefore$ Median = 30–40

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$

$$\Rightarrow \quad 31 = 30 + \left[\frac{20 - (11 + x)}{y}\right] \times 10$$

$$\Rightarrow \quad 1 = \frac{(9 - x) \times 10}{y}$$

$\Rightarrow \quad y = 90 - 10x$

$10x + y = 90 \quad ...(ii)$ **[1]**

On solving eqn. (i) and (ii), we get

$x = 8$ and $y = 10$ **[1]**

OR

Modal class = 60 – 80 **[1]**

$$\therefore \quad Z = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Here, $l = 60$, $f_1 = 29$, $f_0 = 21$, $f_2 = 17$ and $h = 20$

$$\text{Mode} = 60 + \frac{29 - 21}{2 \times 29 - 21 - 17} \times 20$$

$$= 60 + \frac{8}{58 - 38} \times 20$$

$= 60 + 8 = 68$ **[1]**

Empirical relationship = Mode = 3 median – 2 mean

Mode = 68 and mean = 53 (given)

3 median = mode + 2 mean **[1]**

3 median = $68 + 2 \times 53$

$$\text{Median} = \frac{174}{3} = 58$$

Hence Median = 58. **[1]**

MATHEMATICS (STANDARD)

SOLUTIONS
SAMPLE QUESTION PAPER 7

CBSE Class X Examination

Section 'A'

1. (*a*)	**2.** (*a*)	**3.** (*b*)	
4. (*b*)	**5.** (*b*)	**6.** (*a*)	**7.** (*b*)
8. (*b*)	**9.** (*d*)	**10.** (*b*)	

Fill in the blanks

11. $k = -\frac{1}{3}$

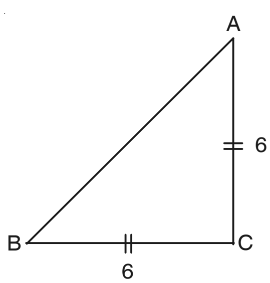
Explanation

If three points are collinear, then area of triangle formed by them is zero.

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$O = \frac{1}{2}[k(3k - 1) + 3k(1 - 2k) + 3(2k - 3k)] = 0$$

$$O = 3k^2 - k + 3k - 6k^2 - 3k$$



$$-3k^2 - k = 0$$

$$-k[3k + 1] = 0$$

$$\Rightarrow \quad k = -\frac{1}{3}$$

12. $AB = 6\sqrt{2}$ cm

Explanation

$AC = BC$ (Isosceles Δ) Applying Pythagoras theorem

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = B^2 + 6^2$$

$$AB^2 = 36 + 36$$

$$AB^2 = 2 \times 36$$

$$AB = 6\sqrt{2} \text{ cm}$$

13. $\theta = 126°$

Explanation

$$\text{Area of sector} = \frac{7}{20} \text{ Area of circle}$$

$$\frac{\theta}{360} \times \pi r^2 = \frac{7}{20}\pi r^2$$

$$\Rightarrow \quad \theta = 126°$$

14. $\frac{16}{9}$

Explanation

$$\text{Given:} \quad \frac{V_1}{V_2} = \frac{64}{27}$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{4}{3}\right)^3$$

$$\left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3$$

$$\therefore \quad \frac{r_1}{r_2} = \frac{4}{3}$$

$$\frac{s_1}{s_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

15. 0

Explanation

cosec (75 + θ) – sec (15 – θ)

= cosec (90 – 15 + θ) – sec (15 – θ)

= cosec [90 – (15 – θ)] – sec (15 – θ)

= sec (15 – θ) – sec (15 – θ)

= 0

Answer the following.

16. $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}$ $\left[\begin{array}{l}\text{As } 1+\tan^2\theta=\sec^2\theta \\ 1+\cot^2\theta=\text{cosec}^2\theta\end{array}\right]$

$= \dfrac{\sec^2\theta}{\text{cosec}^2\theta}$

$= \dfrac{1}{\cos^2\theta}\times\dfrac{\sin^2\theta}{1}$

$= \tan^2\theta$

17. $S_p = ap^2 + bp$

$S_1 = a(1)^2 + b(1) = a + b$

$S_2 = a(2)^2 + b(2) = 4a + 2b$

Clearly $a_1 = s_1 = a + b$

and $a_1 + a_2 = s_2 = 4a + 2b$

$\therefore$ $a_2 = 4a + 2b - a - b$

$a_2 = 3a + b$

Now common difference $= a_2 - a_1$

$= 3a + b - a - b = 2a$

18. Let $\alpha = -2$ and $\beta = 3$

Sum $\alpha + \beta = -2 + 3 = 1$

Product $\alpha\beta = (-2)(3) = -6$

$\therefore$ Quadratic polynomial $= x^2 - 8x + P = 0$

$\Rightarrow$ $x^2 - x - 6 = 0$

Now comparing $a + 1 = -1$

$\Rightarrow$ $a = -2$

$b = -6$

19. The given equation $7x^2 - 12x + 18 = 0$

Here $a = 7$, $b = -12$ and $c = 18$

Sum of roots $= \dfrac{-b}{a} = \dfrac{12}{7}$

Product or roots $= \dfrac{c}{a} = \dfrac{18}{7}$

$\therefore$ Ratio $= \dfrac{12}{18} = \dfrac{2}{3}$

20. $x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x - 3) - 1(x - 3) = 0$

$(x - 3)(x - 1) = 0$

$x = 3, 1$

Section 'B'

21. $\dfrac{1717}{2^2\times5^3} = \dfrac{1717\times2}{2^3\times5^3} = \dfrac{3434}{(10)^3} = \dfrac{3434}{1000} = 3.434$ **[2]**

22. Given, system of equations is

$2x + 3y = 7$

and $(a - 1)x + (a + 1)y = 3a + 1$

On comparing these equations with

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

we get, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$

and $a_2 = (a - 1)$, $b_2 = (a + 1)$, $c_2 = -(3a + 1)$

For parallel lines (no solution), we have

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2} = \dfrac{2}{a-1} = \dfrac{3}{a+1} \neq \dfrac{-7}{-(3a+1)}$

$\Rightarrow$ $2(a + 1) = 3(a - 1)$

$\Rightarrow$ $2a + 2 = 3a - 3$

$\Rightarrow$ $a = 5$ **[1]**

Also, $\dfrac{3}{a+1} \neq \dfrac{7}{3a+1}$

$\Rightarrow$ $9a + 3 \neq 7a + 7$ $\therefore a = 5$

$2a \neq 4$

$a \neq 2$ **[1]**

23. Given A.P. is –6, –9, –12, –15 ...

Here, $a = -6$,

Common difference $(d) = -9 + 6 = -3$ **[1]**

$\therefore$ 11th term of the given A.P. –6, –9, –12, 15...

$T_n = a + (n - 1)d$

$T_{11} = -6 + (11 - 1)(-3)$

$I = -6 - 30$

$= -36$ **[1]**

24. $PA^2 = PB^2$

$\Rightarrow (x - 5)^2 + (y - 1)^2$ **[1]**

$= (x + 1)^2 + (y - 5)^2$

$\Rightarrow$ $12x = 8y$

$\Rightarrow$ $3x = 2y$ **[1]**

CBSE Topper's Solution, 2017

PA = PB

$\therefore PA^2 = PB^2$

by distance formula,

$(5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$

$\Rightarrow (5-x)^2 + (1-y)^2 = (1+x)^2 + (5-y)^2$

$25 - 10x + x^2 + 1 - 2y + y^2 = 1 + 2x + x^2 + 25 - 10y + y^2$

$-10x - 2y = 2x - 10y$

$8y = 12x$

$4(2y) = 4(3x)$

$\therefore 3x = 2y$

Hence, proved.

25. Here, total number of playing cards = 52

Number of red cards and queens = 26 + 2

= 28 **[½]**

$\therefore$ Number of cards other than red cards and queens = 52 – 28 = 24 **[½]**

Required probability of getting neither a red card nor a queen $= \frac{24}{52} = \frac{6}{13}$ **[1]**

26. Total no. of pens = 144

Defective one = 20

Good ones = 144 – 20 = 124

(i) Probability of purchasing pen $= \frac{124}{144} = \frac{31}{36}$ **[1]**

(ii) Probability of not purchasing pen

$= \frac{24}{144} = \frac{5}{36}$ **[1]**

Section 'C'

27. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 + 5 = 5\,(1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 + 1)$ **[1]**

$= 5\,(1008 + 1)$

$= 5(1009)$ **[1]**

= a composite number

[$\because$ product of two factors] **[1]**

28. Let α, β, γ be the zeroes of the given polynomial $f(x)$, such that

$\alpha + \beta = 0$

sum of zeroes $= \frac{-\text{coefficient of } x^2}{\text{coefficient of } x^3}$

$\Rightarrow \quad \alpha + \beta + \gamma = \frac{-4}{1}$

$\Rightarrow \quad 0 + \gamma = -4 \qquad [\alpha + \beta = 0]$

$\gamma = -4$

Product of zeroes

$= \frac{-\text{constant term}}{\text{coefficient of } x^3} \Rightarrow \alpha\beta\gamma = \frac{-(-36)}{1} = 36$

$\therefore \quad \alpha(-\alpha)(-4) = 36$ **[1]**

$\therefore \quad 4\alpha^2 = 36 \Rightarrow \alpha^2 = 9 \Rightarrow \alpha = \pm 3$

Now, $\alpha + \beta = 0, \ \beta = \pm 3$

Hence zeroes are 3, –3, –4. **[1]**

29. Here

$\frac{1}{2}\{(k+1)(-3+k) + 4\,(-k-1) + 7(4)\} = 6$ **[1]**

$\Rightarrow k^2 - 6k + 9 = 0$ **[1]**

Solving to get $k = 3$ **[1]**

[CBSE Marking Scheme, 2015]

CBSE Topper's Solution, 2015

Given, let vertices of a triangle be

$A(k+1, 1)$, $B(4, -3)$ and $C(7, -k)$

Given, $ar(\Delta ABC) = 6$ sq. units.

Here, $x_1 = k+1$, $y_1 = 1$

$x_2 = 4$, $y_2 = -3$

$x_3 = 7$, $y_3 = -k$

Now, $ar(\Delta ABC) = \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

or, $6 = \frac{1}{2}|(k+1)(-3+k)+4(-k-1)+7(1+3)|$

or, $12 = (k+1)(k-3)-4k-4+28$

or, $12 = k^2-2k-3-4k+24$

or, $k^2-6k+21-12=0$

or, $k^2-6k+9=0$, which is a Quad. eqn.

or, $k^2-3k-3k+9=0$

or, $k(k-3)-3(k-3)=0$

or, $(k-3)(k-3)=0$

or, $k-3=0$ | or, $k-3=0$

or, $k=3$ | or, $k=3$

$\therefore k=3$

OR

Diagonals of a parallelogram bisect each other.

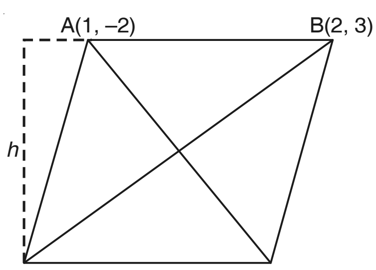
So, mid-point of AC = mid-point of BD

i.e., $\left(\frac{1+k}{2}, \frac{2-2}{2}\right) = \left(\frac{2-4}{2}, \frac{3-3}{2}\right)$

$\Rightarrow \left(\frac{1+k}{2}, 0\right) = (-1, 0)$

i.e., $\frac{(1+k)}{2} = -1$

i.e., $k = -3$ **[1]**



Now ar(ABCD) = 2 Area of ΔABD

$= 2 \times \frac{1}{2} \times [1(6) + 2(-1) - 4(-5)]$

$= 24$ sq. units. **[1]**

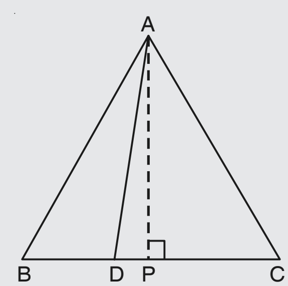
$AB = \sqrt{(2-1)^2+(3+2)^2} = \sqrt{26}$ units

ar(ABCD) = base × height = AB × h

So, $24 = \sqrt{26} \times h$

So, $h = \frac{24}{\sqrt{26}}$ units **[1]**

30. Construction: Draw $AP \perp BC$



[1]

In ΔADP, $AD^2 = AP^2 + DP^2$

$AD^2 = AP^2 + (BP - BD)^2$

$AD^2 = AP^2 + BP^2 + BD^2 - 2(BP)(BD)$ **[1]**

As $AP^2 + BP^2 = AB^2$

$$AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$$

$$AD^2 = \frac{7}{9}AB^2 \ (\because BC = AB)$$

$$9AD^2 = 7AB^2$$ **[1]**

[CBSE Marking Scheme, 2018]

OR

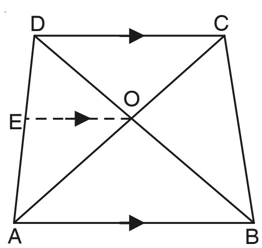
Proof:

In quadrilateral ABCD,

$$\frac{AO}{BO} = \frac{CO}{DO} \quad \text{(Given)}$$

or, $$\frac{AO}{CO} = \frac{BO}{DO} \quad ...(i)$$ **[1]**

Draw EO || AB on



[1]

In ΔABD, EO || AB (By construction)

$$\therefore \quad \frac{AE}{ED} = \frac{BO}{DO} \quad \text{(By BPT) ...(ii)}$$

From eqns. (i) and (ii),

$$\frac{AE}{ED} = \frac{AO}{CO}$$

In ΔADC, $\frac{AE}{ED} = \frac{AO}{CO}$

or, EO || DC (Converse of BPT)

EO || AB (Construction)

∴ AB || DC

or, In quad. ABCD, AB || DC

or, ABCD is a trapezium. **[1]**

Hence Proved.

31. LHS $= \text{cosec}^2\,\theta - \tan^2(90° - \theta)$

$$= \frac{1}{\sin^2\theta} - \frac{\sin^2(90°-\theta)}{\cos^2(90°-\theta)}$$

$$= \frac{1}{\sin^2\theta} - \frac{\sin^2(90°-\theta)}{\sin^2\theta}$$ **[1]**

$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1-\cos^2\theta}{\sin^2\theta}$$ **[1]**

$$= \frac{\sin^2\theta}{\sin^2\theta}$$

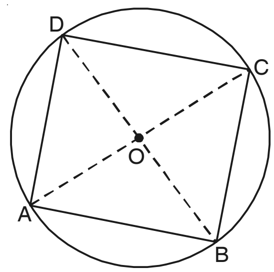
$= 1$

$= \sin^2\theta + \cos^2\theta$

$= \sin^2\theta + \sin^2(90° - \theta)$ **[1]**

= RHS

32. **[1]**



AB = BC = CD = AD

$\Rightarrow$ AC = BD = $2r$

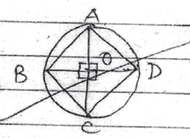
$3.14\,r^2 = 1256 \Rightarrow r = 20$ cm **[1]**

$$\text{Area} = \frac{1}{2}\times 40\times 40 = 800 \text{ cm}^2$$ **[1]**

[CBSE Marking Scheme, 2015]

CBSE Topper's Solution, 2015

In circle with centre O.
ABCD is a rhombus.
As we know that diagonals of a rhombus are perpendicular to each other.



So, AC and BD intersect each other at O.

Now, ar(circle) = 1256 cm^2

or, $\pi r^2 = 1256$

or, $3.14 \times r^2 = 1256 \Rightarrow r^2 = \frac{1256 \times 100}{314}$ (4)

$\Rightarrow r = 2 \times 10$
= 20 cm

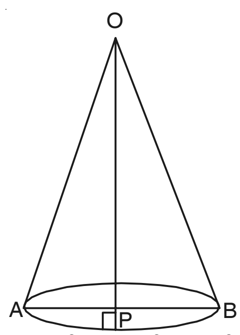
Now, diagonals of rhombus = (2×20) cm
= 40 cm

∴ area of rhombus ABCD = $\frac{1}{2} \times 40 \times 40$ (20)
= 800 cm^2

33. Heap of rice is cone shaped:

Diameter of its base = 24 m

Height of cone = 3.5 m



In ΔOAP $OA^2 = OP^2 + AP^2$

$= (12)^2 + (3.5)^2$

$= 144 + 12.25 = 156.25$

$\therefore$ OA (l) $= \sqrt{156.25} = 12.5$ cm **[1]**

Volume of cone $= \frac{1}{3}\pi r^2 h$ **[1]**

$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5$

$= 22 \times 4 \times 12 \times 0.5$

$= 22 \times 2 \times 12 = 528 \text{ m}^3$ **[1]**

$\therefore$ Volume of cone = 528 m^3

Canvas cloth is required to just cover the heap

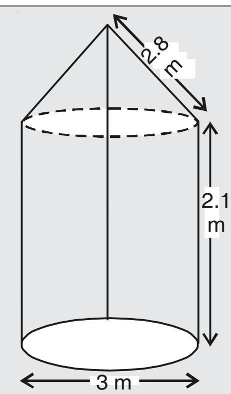
= C.S.A. of cone = πrl

$= \frac{22}{7} \times 12 \times 12.5 = \frac{22 \times 150.0}{7}$

$= \frac{3300}{7}$

Canvas cloth = 471.43 m^2 (Approx) **[1]**

OR



Area of canvas needed

$= 2 \times \frac{22}{7} \times (1.5) \times 2.1 + \frac{22}{7} \times 1.5 \times 2.8$ **[2]**

$= \frac{22}{7}[6.3 + 4.2] = \frac{22}{7} \times 10.5 = 33 \text{ m}^2$ **[1]**

cost = 33 × 500 = ₹ 16500 **[1]**

[CBSE Marking Scheme, 2016]

CBSE Topper's Solution, 2016

Radius of cylinder as well as conical part = $\frac{3}{2}$ m.

Height of cylinder, h = 2.1 m

Slant height of cone, l = 2.8 m.

Total canvas required = $2\pi rh + \pi rl$

$\pi r(2h + l)$

$\Rightarrow \frac{22}{7} \times \frac{3}{2} [4.2 + 2.8] m^2$

$\Rightarrow \frac{22}{7} \times \frac{3}{2} \times 7.0 m^2 = 33 m^2$

Total cost @ ₹500/m² = ₹33 × 500

₹16,500

34.

Height	Frequency	*c.f.*
100–140	22	22
140–180	14	36
180–220	18	54
220–260	16	70
260–300	30	100
	Total	**100**

[2]

Here, $N = 100$

$\Rightarrow$ Median $= \frac{N}{2}$ th term

$= \frac{100}{2} = 50$th term

So, Median Class = 180 – 220

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$ **[1]**

$$= 180 + \left(\frac{50 - 36}{18}\right) \times 40$$

$$= 180 + \frac{14 \times 40}{18}$$

$= 180 + 31.11$

Median $= 211.11$ **[1]**

Section 'D'

35. $(x - 1)^2 + (2x + 1)^2 = 2(2x + 1)(x - 1)$ **[1]**

$\Rightarrow x^2 + 1 - 2x + 4x^2 + 1 + 4x$

$= 4x^2 - 4x + 2x - 2$

$\Rightarrow x^2 + 4x + 4 = 0$ **[1]**

$\Rightarrow (x + 2)^2 = 0$

$\Rightarrow x = -2$ **[1]**

[CBSE Marking Scheme, 2017]

CBSE Topper's Solution, 2017

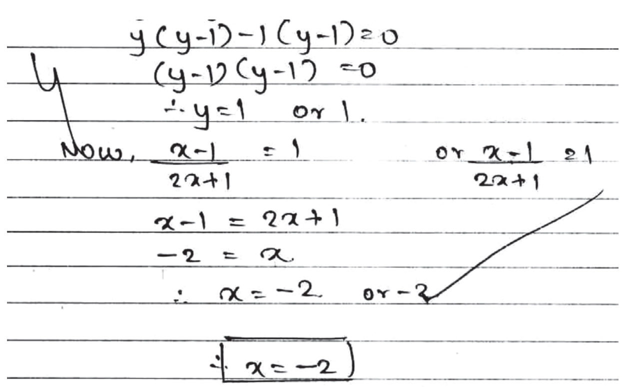
Let $\frac{x-1}{2x+1}$ be y,

$y + \frac{1}{y} = 2$

$y^2 + 1 = 2y$

$y^2 - 2y + 1 = 0$

$y^2 - y - y + 1 = 0$


y(y-1)-1(y-1)=0
(y-1)(y-1) =0
∴ y=1 or 1.
Now, (x-1)/(2x+1) = 1 or (x-1)/(2x+1) = 1
x-1 = 2x+1
-2 = x
∴ x = -2 or -2
∴ x = -2

OR

Let the original speed be x km/h.

Then, reduced speed = $(x - 400)$ km/h

The duration of flight at original speed

$$= \left(\frac{6000}{x}\right)\text{hours}$$

$$\left[\because \text{time} = \frac{\text{distance}}{\text{speed}}\right]$$

The duration of flight at reduced speed

$$= \left(\frac{6000}{x-400}\right)\text{hours} \qquad ...(1)$$

Since, the difference between these two duration is $\frac{1}{2}$ hours.

$$\therefore \quad \frac{6000}{x-400} - \frac{6000}{x} = \frac{1}{2} \quad \textbf{[1]}$$

$$\Rightarrow \quad \frac{6000x - 6000(x-400)}{x(x-400)} = \frac{1}{12000}$$

$$\Rightarrow \quad \frac{6000\{x-(x-400)\}}{x(x-400)} = \frac{1}{12000}$$

$$\Rightarrow \quad \frac{x-x+400}{x(x-400)} = \frac{1}{12000}$$

$$\Rightarrow \quad \frac{400}{x(x-400)} = \frac{1}{12000}$$

$\Rightarrow \quad x(x - 400) = 4800000$

$\Rightarrow \quad x^2 - 400x - 4800000 = 0$ **[1]**

$\Rightarrow x^2 - 2400x + 2000x - 4800000 = 0$

[by factorsation]

$\Rightarrow x(x - 2400) + 2000(x - 2400) = 0$

$\Rightarrow \quad (x - 2400)(x + 2000) = 0$

$\Rightarrow \quad x - 2400 = 0$ or $x + 2000 = 0$

$\Rightarrow \quad x = 2400$ or -2000 **[1]**

Since, the speed of aircraft cannot be negative

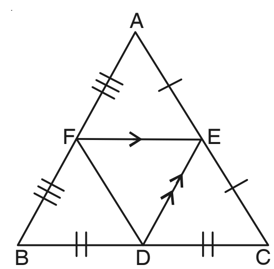
So, $\quad x = 2400$

Hence, the original speed is 2400 km/h.

$\therefore$ Original duration of the flight

$$= \frac{6000}{2400} = 2\frac{1}{2}\text{hours} \quad \left[\because \text{time} = \frac{\text{distance}}{\text{speed}}\right] \quad \textbf{[1]}$$

36. In ΔABC, Given that F, E and D are the mid-points of AB, AC and BC respectively.



[1]

Hence, FE ∥ BC, DE ∥ AB and DF ∥ AC

By mid-point theorem.

If DE ∥ BA

then DE ∥ BF

and if FE ∥ BC

then FE ∥ BD

$\therefore$ FEDB is parallelogram in which DF is diagonal and a diagonal of Parallelogram divides it into two equal areas.

Hence ar (ΔBDF) = ar (ΔDEF) ...(*i*) **[½]**

Similarly ar (ΔCDE) = ar (ΔDEF) ...(*ii*) **[½]**

or (ΔAFE) = ar (ΔDEF) ...(*iii*) **[½]**

or (ΔDEF) = ar (ΔDEF) ...(*iv*) **[½]**

On adding eqns. (*i*), (*ii*), (*iii*) and (*iv*),

$ar(\Delta BDF) + ar(\Delta CDE) + ar(\Delta AFE) + ar(\Delta DEF)$

$= 4ar(\Delta DEF)$

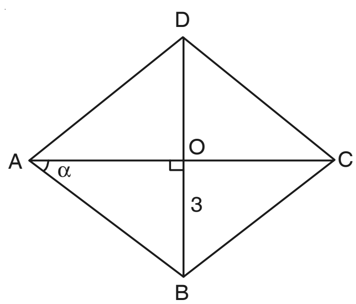
or, $ar(\Delta ABC) = 4ar(\Delta DEF)$

$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$ **[1]**

OR

Given : $\cos \alpha = \frac{2}{3}$

and OB = cm



In ΔAOB, $\cos \alpha = \frac{2}{3} = \frac{AO}{AB}$

Let OA = $2x$ and AB = $3x$

In ΔAOB,

$AB^2 = AO^2 + OB^2$

or, $(3x)^2 = (2x)^2 + (3)^2$

or, $9x^2 = 4x^2 + 9$

or, $5x^2 = 9$ **[1]**

$\therefore$ $x = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$

Hence, $OA = 2x = 2\left(\frac{3}{\sqrt{5}}\right) = \frac{6}{\sqrt{5}}$ cm **[1]**

and $AB = 3x = 3\left(\frac{3}{\sqrt{5}}\right) = \frac{9}{\sqrt{5}}$ cm **[1]**

So diagonal BD = $2 \times OB = 2 \times 3 = 6$ cm

and AC = 2AO

$= 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}}$ cm **[1]**

37. Given : A circle of radius 4 cm.

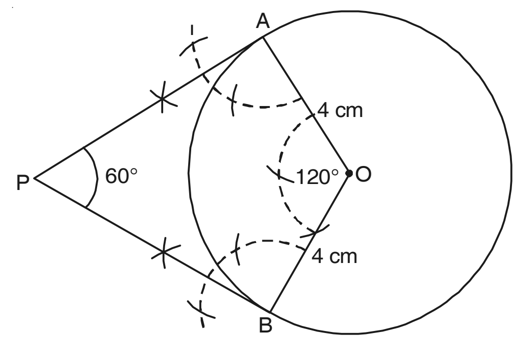
Required : A pair of tangents, such that angle between them is 60º.

Step of Construction :

1.Draw a circle of radius 4 cm with centre *O*.

2.Draw any radius OA.

3.Draw another radius OB, such that $\angle AOB = 120º$. **[2]**



[2]

[$\therefore \angle P + \angle O = 180º$

$\Rightarrow 60º + \angle O = 180º$

$\Rightarrow \angle O = 120º$]

4. At point A, draw AP perpendicular to OA.

5. At point B, draw BP perpendicular to OB and let the perpendiculars meet at P, such that $\angle P = 60º$.

Thus, PA and PB are the required tangents.

38. Let the first average speed of the bus be x km/h.

$\therefore$ $\frac{75}{x} + \frac{90}{x+10} = 3$

$\Rightarrow 75x + 750 + 90x = 3(x^2 + 10x)$ **[1]**

$\Rightarrow x^2 - 45x - 250 = 0$ **[1]**

Solving to get $x = 50$ **[1]**

$\therefore$ Speed = 50 km/h. **[1]**

[CBSE Marking Scheme, 2017]

CBSE Topper's Solution, 2015

Let the avg. speed for a dist. of 75 km = x km/hr

Then, time taken to cover 75 km = $\left[\frac{75}{x}\right]$ hrs

Now, speed for the next 90 km = $(x+10)$ km/hr

Time taken to cover 90 km = $\left[\frac{90}{x+10}\right]$ hrs.

A/Q

$$\frac{75}{x} + \frac{90}{x+10} = 3$$

or, $15\left[\frac{5}{x} + \frac{6}{x+10}\right] = 3$

or, $\frac{5(x+10)+6x}{x^2+10x} = \frac{1}{5}$

or, $5x+50+6x = \frac{x^2+10x}{5}$

or, $(11x+50)5 = x^2+10x$

or, $x^2+10x-55x-250=0$

or, $x^2-45x-250=0$, which is a Quad. eqn

or, $x^2-50x+5x-250=0$

or, $x(x-50)+5(x-50)=0$

or, $(x-50)(x+5)=0$

or, $x-50=0$ | or, $x+5=0$

or, $x=50$ | or, $x=-5$ (invalid)

∴ First speed = $x = 50$ km/hr

39. Volume of rain water on the roof = Volume of cylindrical tank **[1]**

i.e., $22 \times 20 \times h = \frac{22}{7} \times 1 \times 1 \times 3.5 \quad \Rightarrow h = \frac{1}{40}\text{ m} = 2.5\text{ cm}$ **[2]**

Water conservation must be encouraged or views relevant to it. **[1]**

CBSE Solution's Answer, 2015

radius of cylindrical tank = $\frac{2}{2}$ = 1 m.

its height = 3.5 m. = $\frac{35}{10}$ m

Let the height of water on roof be h.

Volume of water on roof = volume of water in tank.

$lbh = \pi r^2 h'$

$22 \times 20 \times h = \frac{22}{7} \times \frac{35}{10} \times 1 \times 1$

$h = \frac{22}{2} \times \frac{1}{22} \times \frac{1}{20} = \frac{1}{40}$ m

∴ $h = \frac{1}{40}$ m $= \frac{1}{40} \times 100$ cm

$= 2.5$ cm

So, the rainfall is 2.5 cm

Views on water conservation:

a) It is important practice in today's era of irrational water consumption and pollution. It should be practised at municipal level at all places.

b) It can be done by many simple ways even at domestic level.

c) Doing it is a sign of environmental consciousness.

d) Some methods of water conservation are rooftop/surface water harvesting, building small earthen dams, etc.

e) This conserved water helps refill underground water bodies and so, all must practise water conservation for sustainable development.

40. Find the median of the students and how can get the median graphically

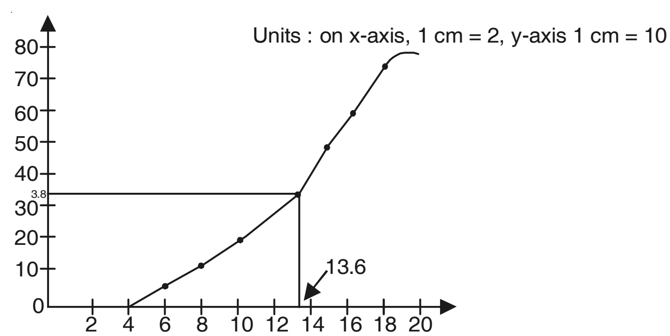
Age of student	C.I.	*c.f.*
Less than 6	4–6	2
Less than 8	6–8	6
Less than 10	8–10	12
Less than 12	10–12	22
Less than 14	12–14	42
Less than 16	14–16	67
Less than 18	16–18	76
		N = 76

[2]

$$\text{Median} = \frac{N}{2}\text{th term}$$

$$= \frac{76}{2} = 38\text{th term}$$

Median class = 12 – 14



[2]

Hence Medium = 13.6

OR

Let assumed mean, $A = 649.5$

Life time (in hrs)	x_i	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
399.5–499.5	449.5	–2	24	–48
499.5–599.5	549.5	–1	47	–47
599.5–699.5	649.5 = a	0	39	0
699.5–799.5	749.5	1	42	42
799.5–899.5	849.5	2	34	68
899.5–999.5	949.5	3	14	42
Total			$\Sigma f_i = 200$	$\Sigma f_i u_i = 57$

[2]

Mean, $x = a + \left(\frac{\sum f_i u_i}{\sum f_i} \times h\right) = 649.5 + \frac{57}{200} \times 100 = 649.5 + 28.5 = 678$ [1]

Hence, Mean life time of a bulb is 678 hours. [1]

MATHEMATICS (STANDARD)

SOLUTIONS
SAMPLE QUESTION PAPER 8

CBSE Class X Examination

Section 'A'

1. (*c*) **2.** (*b*) **3.** (*a*) **4.** (*c*)
5. (*c*) **6.** (*d*) **7.** (*b*) **8.** (*c*)
9. (*c*) **10.** (*d*)

Fill in the blanks

11. $a = -\frac{3}{2}$

Explanation

The given equation is $5x^2 - \frac{7}{2}x + 2a = 0$

putting $x = \frac{1}{2}$

$$5\left(-\frac{1}{2}\right)^2 - \frac{7}{2}\left(-\frac{1}{2}\right) + 2a = 0$$

$$\frac{5}{4} + \frac{7}{4} + 2a = 0$$

$$a = -\frac{3}{2}$$

12. $x + 63$

Explanation

Here $a = x - 7$

$d = (x - 2) - (x - 7) = 5$

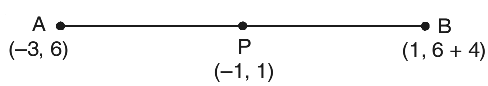
$\therefore\ a_{15} = a + 14d$

$= x - 7 + 14(5)$

$= x + 63$

13. $b = -1$

Explanation



Applying mid-point formula for Y coordinate

$$\frac{b + b + 4}{2} = 1$$

$2b + 4 = 2$

$2b = -2$

$b = -1$

14. $QB = 3$ cm

Explanation

$\Delta AOP \sim \Delta BOQ$ (AA)

$$\therefore \quad \frac{OA}{AP} = \frac{BO}{QB}$$

$$\Rightarrow \quad \frac{6}{4} = \frac{4.5}{QB}$$

$\Rightarrow \quad QB = 3$ cm

15. 2

Explanation

$$\frac{\sin 25^\circ}{\cos 65^\circ} + \frac{\tan 23^\circ}{\cot 67^\circ}$$

$$= \frac{\sin(90^\circ - 65)}{\cos 65^\circ} + \frac{\tan(90^\circ - 67^\circ)}{\cot(67^\circ)}$$

$$= \frac{\cos 65^\circ}{\cos 65^\circ} + \frac{\cot 67^\circ}{\cot 67^\circ}$$

$= 1 + 1 = 2$

Answer the following.

16. $\sin(45^\circ + Q) - \cos(45^\circ - Q)$

$= \cos[90 - (45^\circ - Q)] - \cos(45^\circ - Q) = 0$

$= \cos(45^\circ - Q) - \text{co}(45^\circ - Q)$

$= 0$

17. Let one root be α, then other root will be $-\alpha$,then other root will be $-\alpha$.

Given equation is $3x^2 - px + 5 = 0$

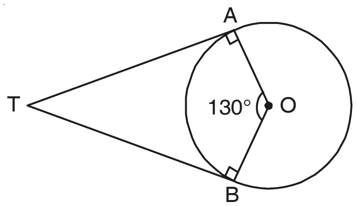
Here $a = 3$, $b = -p$ and $c = 5$

Sum of roots $\alpha + \beta = {-b}/{a}$

$$\therefore \quad \alpha + (-\alpha) = \frac{-(-p)}{3}$$

$$0 = \frac{P}{3}$$

$\therefore \quad P = 0$

18. 

In quad. $AOBT$, $\angle A = B = 90°$

Also $\angle A + \angle O + \angle B + \angle T = 360°$

$90° + 130° + 90° + \angle T = 360°$

$\therefore \quad \angle T = 50°$

19. In ΔPAO, $(OP)^2 = (12)^2 + (5)^2$

$\therefore \quad (OP)^2 = 169$

$\therefore \quad OP = 13$

Now in ΔPOB

$(PB)^2 = (13)^2 - (3)^2$

$= 169 - 9$

$= 160$

$\therefore PB = \sqrt{160} = 4\sqrt{10}$ cm

20. $PA = PB$ (tangents from an external point)

$\therefore \angle PAB = \angle PBA = x$ (isosceles triangle)

$\Rightarrow$ In ΔABP

$x + x + 60° = 180°$ (Angle sum property)

$\Rightarrow x = 60°$

$\therefore \Delta\ ABP$ is an equilateral triangle.

$\therefore AB = 5$ cm

Section 'B'

21. 366 days = 52 weeks + 2 days **[1]**

2 days can be MT, TW, WT, TF, FS, SS, SM = 7

$\Rightarrow \quad P = \frac{2}{7}$ **[1]**

(CBSE Marking Scheme, 2012)

22. Total number of outomes = 36

Favourable outcomes are (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) = 5 **[1]**

Required probability = $\frac{5}{36}$ **[1]**

(CBSE Marking Scheme, 2012)

23. $3 \times 12 \times 101 + 4 = 4\,(3 \times 3 \times 101 + 1)$

$= 4\,(909 + 1)$ **[1]**

$= 2 \times 2 \times 2 \times 5 \times 7 \times 13$

= a composite number **[1]**

[$\because$ Product of more than two prime factors]

(CBSE Marking Scheme, 2015)

24. Given, $kx - 4y - 3 = 0$

and $6x - 12y - 9 = 0$

where, $a_1 = k$, $b_1 = -4$, $c_1 = -3$

$a_2 = 6$, $b_2 = -12$, $c_2 = -9$

condition for infinite solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ **[1]**

$$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9}$$

Hence, $k = 2$ **[1]**

25. Let the first term be a and common difference d given

$5a_5 = 8a_8$

$5\,(a + 4d) = 8\,(a + 7d)$

$5a + 20d = 8a + 56d$

$3a + 36d = 0$ **[1]**

$3\,(a + 12d) = 0$

$a + 12d = 0$

$a_{13} = 0$ **[1]**

(CBSE Marking Scheme, 2012)

26. Let the point on y-axis be P(0, y)

and AP : PB = K : 1

Therefore $\frac{5-K}{K+1} = 0$ gives K = 5

Hence required ratio is 5 : 1 **[1]**

$$y = \frac{-4(5)-6}{6} = \frac{-13}{3}$$

Hence point on y-axis is $\left(0, \frac{-13}{3}\right)$. **[1]**

Section 'C'

27. P (x, y), A (6, 2), B (–2, 6)

PA = PB or, $PA^2 = PB^2$

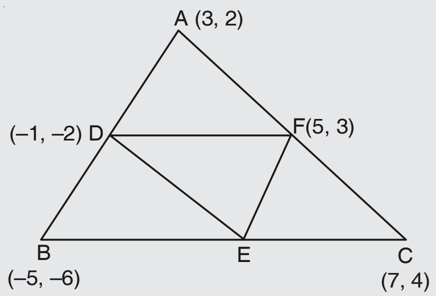
$(x - 6)^2 + (y - 2)^2 = (x + 2)^2 + (y - 6)^2$ **[1]**

or $x^2 - 12x + 36 + y^2 - 4y + 4 = x^2 + 4x + 4 + y^2 - 12y + 36$

$$-12x - 4y = 4x - 12y \quad \textbf{[1]}$$
$$12y - 4y = 4x + 12x$$
$$8y = 16x$$
$$y = 2x \quad \textbf{[1]}$$

(CBSE Marking Scheme, 2015)

OR



$$\text{Mid point of BA} = \frac{3+(-5)}{2}$$
$$= -1 \text{ and } \frac{2-6}{2} = -2$$
$$D = (-1, -2)$$
$$\text{Mid points of BC} = \frac{-5+7}{2}$$
$$= 1 \text{ and } \frac{-6+4}{2} = -1$$
$$E = (1, -1)$$

Mid points of

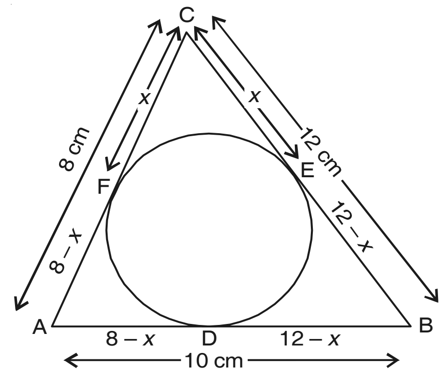
$$CA = \frac{7+3}{2} = 5 \text{ and } \frac{4+2}{2} = 3$$
$$F = (5, 3)$$

Now, Area of ΔDEF

$$= \frac{1}{2}\left[-1(-1-3)+1(3+2)+5(-2+1)\right]$$
$$= \frac{1}{2}[4+5-5]$$
$$= 2 \text{ unit} \quad \textbf{[1+1+1]}$$

(CBSE Marking Scheme, 2012)

28. AC = 8 cm, AB = 10 cm & BC = 12 cm



[1]

Let CF = x

$$CF = EC = x$$
$$AF = 8 - x = AD$$
$$BE = 12 - x = BD \quad \textbf{[1]}$$

or, $8 - x + 12 - x = 10$

$$20 - 2x = 10$$
$$-2x = -10, \text{ or } x = 5$$

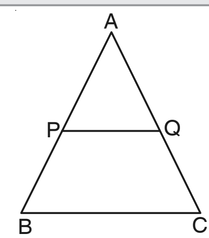
∴ AD = 3 cm

BE = 7 cm

CF = 5 cm **[1]**

(CBSE Marking Scheme, 2012)

29.



$$\frac{AP}{AB} = \frac{3.5}{10.5} = \frac{1}{3} \quad \textbf{[1]}$$

or
$$\frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$$

In ΔABC, $\frac{AP}{AB} = \frac{AQ}{AC}$ and ∠A is common

or ΔAPQ ~ ΔABC

(By SAS) **[1]**

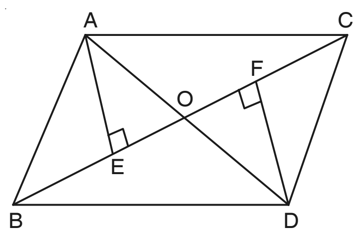
$$\frac{AP}{AB} = \frac{PQ}{BC}$$

or $\frac{1}{3} = \frac{4.5}{BC}$

or $BC = 13.5$ cm **[1]**

OR

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$



[1]

Construction : Draw AE ⊥ BC and DF ⊥ BC

Proof : In ΔAOE and ΔDOF,

$\angle AOE = \angle DOF$

(vertically OPP. angles)

$\angle AEO = \angle DFO = 90°$

(Construction)

$\Delta AOE \sim \Delta DOF$

(By AA similarity)

$\therefore \frac{AO}{DO} = \frac{AE}{DF}$ ...(*i*) **[1]**

Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}\times BC\times AE}{\frac{1}{2}\times BC\times DF}$

$= \frac{AE}{DF}$

$= \frac{AO}{DO}$ [From eq. (*i*)] **[1]**

Hence proved.

30.

$$\begin{array}{r} x^2+2x\,\big)\,x^3+5x^2+7x+3\,\big(\,x+3 \\ x^3+2x^2 \\ - \\ \hline 3x^2+7x+3 \\ 3x^2-6x \\ - \\ \hline \end{array}$$

Remainder = $x + 3$ **[2]**

Hence, $-(x + 3)$ must be added **[1]**

(CBSE Marking Scheme, 2016)

31. Let $3+\sqrt{5}$ be a rational number.

$\therefore \quad 3+\sqrt{5} = \frac{p}{q}, q \neq 0$ **[1]**

$3+\sqrt{5} = \frac{p}{q}$

$\Rightarrow \quad \sqrt{5} = \frac{p}{q} - 3$

$\Rightarrow \quad \sqrt{5} = \frac{p-3q}{q}$

$\sqrt{5}$ is irrational and $\frac{p-3q}{q}$ is rational **[1]**

But rational number connot be equal to an irrational number.

$\therefore \quad 3+\sqrt{5}$ is an irrational number. **[1]**

32. $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\sin\theta = \cos\theta(\sqrt{2}-1)$ **[1]**

$\sin\theta = \frac{\cos\theta(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)}$

$\sin\theta = \frac{\cos\theta(2-1)}{\sqrt{2}+1}$ **[1]**

$(\sqrt{2}+1)\sin\theta = \cos\theta$

$\sqrt{2}\sin\theta + \sin\theta = \cos\theta$

$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ **[1]**

OR

$LHS = \frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A}$

$= \frac{\cos A}{1+\frac{\sin A}{\cos A}} - \frac{\sin A}{1+\frac{\cos A}{\sin A}}$ **[1]**

$= \frac{\cos^2 A}{\cos A+\sin A} - \frac{\sin^2 A}{\sin A+\cos A}$

$= \frac{\cos^2 A - \sin^2 A}{(\sin A+\cos A)}$ **[1]**

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A + \cos A}$$

$= \cos A - \sin A$ **[1]**

= RHS **Hence Proved.**

33. Volume of water in cylinder

$= \text{volume of cylinder}$

$= \pi r^2 h$

$= \pi \times (60)^2 \times 180$

$= 648000\pi \text{ cm}^3$

water displaced on dropping cone, volume of solid cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times (30)^2 \times 60$

$= 18000\pi \text{ cm}^3$

volume of water left in cylinder = volume of cylinder – volume of cone

$= 648000\pi - 18000\pi$

$= 630000\pi \text{ cm}^3$

$= \frac{630000 \times 22}{1000000 \times 7} \text{ m}^3$

$= 1.98 \text{ m}^3$

[3] (CBSE Marking Scheme, 2015)

OR

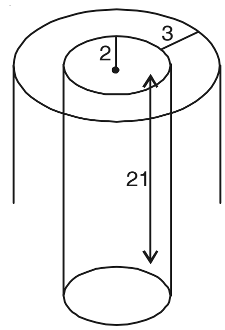
Diameter of earth dug out = 4 m

Radius of earth dug out = 2 m

Depth of the earth = 21

Volume of earth $= \pi r^2$

$= \frac{22}{7} \times 2 \times 2 \times 21 = 264 \text{ m}^2$ **[1]**



Width of embankment = 3 m

Outer radius of ring = 2 + 3 = 5 m

Let the height of embankment = h

∴ Volume of embankement

$\pi (R^2 - r^2)\, h = 264$ **[1]**

$\frac{22}{7}\left[(5)^2 - (2)^2\right] h = 264$

$\frac{22}{7} \times [25 - 4] h = 264$

$\frac{22}{7} \times 21 \times h = 264$

$h = \frac{264 \times 7}{22 \times 21}$

$h = 4$ m. **[1]**

34.

Classes	**Frequency f_i**	**Mid-point x_i**	**$f_i x_i$**
0–20	6	10	60
20–40	8	30	240
40–60	10	50	500
60–80	12	70	840
80–100	8	90	720
100–120	6	110	660
	$\Sigma f_i = 50$		**$\Sigma f_i x_i = 3020$**

[2]

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3020}{50}$$

$= 60.4$ **[1]**

Section 'D'

35. Let the speed while going be x km/h

$\therefore$ Speed while returning = $(x + 10)$ km/h

According to question,

$$\frac{150}{x} - \frac{150}{x+10} = \frac{5}{2}$$

$$\Rightarrow \frac{150(x+10) - x(150)}{x(x+10)} = \frac{5}{2}$$

$$\Rightarrow \frac{150x + 1500 - 150x}{x^2 + 10x} = \frac{5}{2}$$

$$\Rightarrow 5(x^2 + 10x) = 2 \times 1500$$

$$\Rightarrow x^2 + 10x = 600$$

$$\Rightarrow x^2 + 10x - 600 = 0$$

$$\Rightarrow (x + 30)(x - 20) = 0$$

or $x = 20$

$\therefore$ Speed while going = 20 km/h

and speed while returning = 20 + 10 = 30 km/h

[3] [CBSE Marking Scheme, 2016]

OR

Here $x = -2$ is the root of the equation

$$3x^2 + 7x + p = 0$$

or $3(-2)^2 + 7(-2) + p = 0$

$\Rightarrow p = 2$

Root of the equation $x^2 + 4kx + k^2 - k + 2 = 0$ are equal.

or $16k^2 - 4(k^2 - k + 2) = 0$

$$16k^2 - 4k^2 + 4k - 8 = 0$$

$$12k^2 + 4k - 8 = 0$$

$$3k^2 + k - 2 = 0$$

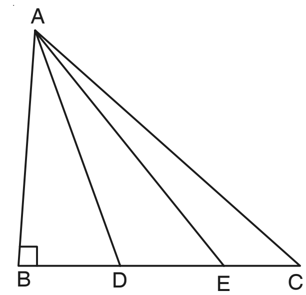
$$(3k - 2)(k + 1) = 0$$

$$k = \frac{2}{3}, -1$$

Hence, roots = $\frac{2}{3}, -1$

[3] [CBSE Marking Scheme, 2015]

36.



[1]

Let BD = DE = EC be x

and BE = $2x$

BC = $3x$ **[1]**

Now in ΔABE, $AE^2 = AB^2 + BE^2$

$= AB^2 + 4x^2$ ...(*i*)

In ΔABC, $AC^2 = AB^2 + BC^2 = AB + 9x^2$

In ΔADB, $AD^2 = AB^2 + BD^2 = AB^2 + x^2$

Now, on multiplying (*i*) by 8

$8\,AE^2 = 8\,AB^2 + 32x^2$ ...(*ii*) **[1]**

$3\,AC^2 + 5\,AD^2 = 3(AB^2 + 9x^2) + 5(AB^2 + x^2)$

$= 3\,AB^2 + 27x^2 + 5\,AB^2 + 5x^2$

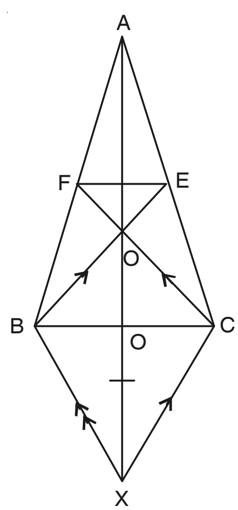
$= 8\,AB^2 + 32x^2 = 8\,AB^2 + 4x^2$ ...(*iii*)

$\therefore 3\,AC^2 + 5\,AD^2 = 8\,AE^2$ **[1]**

[From eq. (*ii*) & (*iii*)]

Hence Proved.

OR



[1]

BC and OX bisect each other.

So, BXCO is a parallelogram, BE ∥ XC and BX ∥ CF

(*i*) In ΔABX, by B.P.T.

$$\frac{AF}{FB} = \frac{AO}{OX} \quad ...(i)$$

In ΔAXC, $$\frac{AE}{EC} = \frac{AO}{OX} \quad ...(ii)$$ **[1]**

(*i*) and (*ii*) give

$$\frac{AF}{FB} = \frac{AE}{EC}$$

So by converse of B.P.T.,

$$EF \parallel BC$$

(*ii*) $$\frac{OX}{OA} = \frac{FB}{AF}$$ **[1]**

Adding 1 on both sides

$$\frac{AX}{OA} = \frac{AB}{AF}$$

or $$OA : AX = AF : AB$$ **[1]**

Hence Proved.

37. Steps of Construction :

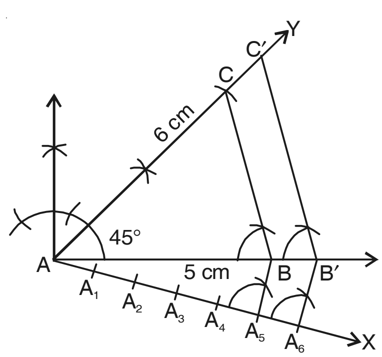
(i) Draw a line segment AB = 5cm

(ii) At A make ∠BAY = 45°

(iii) Take A as centre and radius AC = 6 cm, draw an arc cutting AY at C. **[1]**

(iv) Join BC to obtain the ΔABC.

(v) Draw any ray AX making an acute angle with AB on the side opposite to the vertex C. **[1]**



(vi) Locate 6 points (the greater of 6 and 5 in A_1, A_2, A_3, A_4, A_5 and A_6) on AX, such that

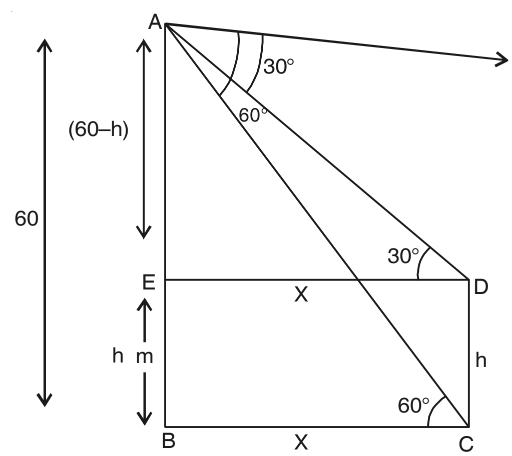
$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4 = A_5 = A_5A_6$ **[1]**

(vii) Join the 5th point, being the smaller of 5 and 6 in 6/5 to B and draw a line segment througr A_6 parallel A_5B intersecting the extended line segment AB at B′.

(viii) Draw a line through B parallel to BC intersecting the extended line segment AC at C′.

Then AB′C′ is the required triangle. **[1]**

38. Let AB is a building 60 m high and is a tower *h* meter high. Angle of depressions of top and bettom are given 30º and 60º respectively



[1]

Let $$DC = EB = h \text{ m and}$$
$$BC = x$$
$$\Rightarrow \quad AE = (60 - h)\text{m}$$

In ΔAED, $$\frac{60-h}{ED} = \tan 30^\circ$$ **[½]**

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{60-h}{BC}$$

$$\Rightarrow \quad \sqrt{3}(60-h) = x \quad ...(i)$$ **[½]**

In ΔABC, $$\frac{60}{x} = \tan 60^\circ$$

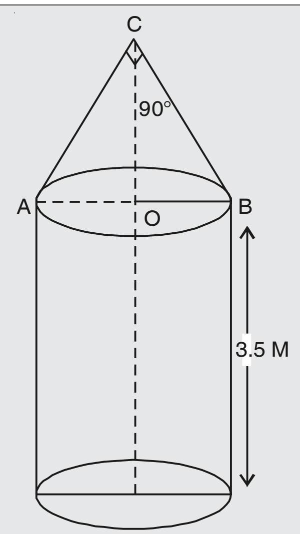
$$60 = \sqrt{3}x \quad ...(ii)$$ **[1]**

Putting the value of x from equation (i) in equation (ii), we get

$$60 = \sqrt{3}\times\sqrt{3}(60-h)$$
$$60 = 3 \times (60 - h)$$
$$20 = 60 - h$$
$$h = 40 \text{ m}$$

Hence, Height of tower = 40m. **[1]**

39.



Since, $\angle C = 90°$

$AC = BC = x$ (say)

$\therefore$ $AB^2 = AC^2 + BC^2$

$AB^2 = x^2 + x^2$

$$2x^2 = \left(2\sqrt{2}\right)^2$$

or, $x = \sqrt{2}$ and $r = \sqrt{2}$ m

$\therefore$ Slant height of conical portion = 2m

Total surface area of toy

$$= 2\pi rh + \pi r^2 + \pi rl$$
$$= \pi r[2h + r + l]$$
$$= \pi\sqrt{2}\left[2\times3.5+\sqrt{2}+2\right]$$
$$= \pi\sqrt{2}\left[7+\sqrt{2}+2\right]$$
$$= \pi\sqrt{2}\left[9+\sqrt{2}\right]$$
$$= \pi\left[9\sqrt{2}+2\right]\text{m}^2$$

[4] [CBSE Marking Scheme, 2012]

40.

Classes	***f***	***c.f.***
5–10	2	2
10–15	12	14
15–20	2	16
20–25	4	20
25–30	3	23
30–35	4	27
35–40	3	30

[2]

Total $\sum f = 30 = N$

Since, Median = $\dfrac{N}{2}$th term

$= \dfrac{30}{2} = 15$th term **[½]**

Median class = 15–20 **[½]**

$$\text{Medain} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right)\times h$$

$l = 15$, $N = 30$, $c.f. = 14$, $f = 2$ & $h = 5$

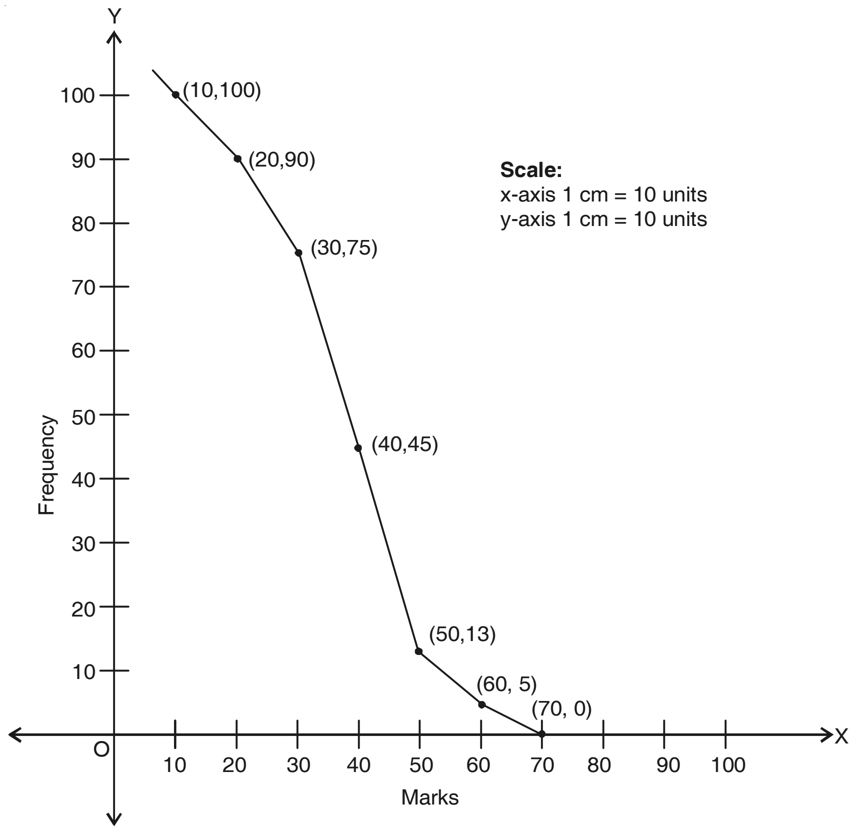
$$\text{Median} = 15+\left(\frac{15-14}{2}\right)\times 5$$

$= 15 + 2.5 = 17.5$ **[1]**

OR

x	***y***
More than 10	100
More than 20	90
More than 30	75
More than 40	45
More than 50	13
More than 60	5
More than 70	0

[2]


Y
X
O
100
90
80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100
Frequency
Marks
(10,100)
(20,90)
(30,75)
(40,45)
(50,13)
(60, 5)
(70, 0)
Scale:
x-axis 1 cm = 10 units
y-axis 1 cm = 10 units

MATHEMATICS (STANDARD)

SOLUTIONS SAMPLE QUESTION PAPER 9

CBSE Class X Examination

Section 'A'

1. (*a*)	**2.** (*b*)	**3.** (*c*)	**4.** (*c*)
5. (*a*)	**6.** (*a*)	**7.** (*b*)	**8.** (*a*)
9. (*d*)	**10.** (*c*)		

Fill in the blanks

11. $k = 3$

Explanation

Let one zero be α, then other will be $\frac{1}{\alpha}$

Given equation is $3x^2 + 8x + k = 0$

Here $a = 3, b = 8$ and $c = k$

product of zeroes $= \alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$

$\Rightarrow \quad 1 = \frac{k}{3}$

$\Rightarrow \quad k = 3$

12. –2/3

Explanation

Given equation is $kx^2 + 2x + 3k = 0$

Here $a = k$, $b = 2$ and $c = 3k$

Sum of zeroes = product of zeroes

$$\frac{-b}{a} = \frac{c}{a}$$

$$\frac{-2}{k} = \frac{3k}{k} \quad \text{or} \quad k = -2/3$$

13. $a = 3$

Explanation

Applying mid-point formula for Y-coordinate

$$\frac{7+a}{2} = 5$$

$\Rightarrow \quad a = 3$

14. 10/9

Explanation

Given equation is

$$x^2 - 2(1 + 3k)x + 7(3 + 2k) = 0$$

Here $a = 1$, $b = -2(1 + 3k)$ and $c = 7(3 + 2k)$

For equal roots,

$$b^2 - 4ac = 0$$

$$[-2(1 + 3k)]^2 - 4 \times 1 \times 7(3 + 2k) = 0$$

$$4(1 + 3k)^2 - 4 \times (21 + 14k) = 0$$

$$4[1 + 9k^2 + 6k - 21 - 14k] = 0$$

$$9k^2 - 8k - 20 = 0$$

$$9k^2 - 18k + 10k - 20 = 0$$

$$9k(k - 2) + 10(k - 2) = 0$$

$$(k - 2)(9k + 10) = 0$$

$\Rightarrow \quad k = 2$ or 10/9

15. 40

Explanation

Let's reverse the A.P. and then find 4^{th} term from the begining.

49, 46, ... –5, –8, –11.

Here $a = 49$ and $d = -3$

$\therefore \quad a_4 = a + 3d$

$= 49 + 3(-3)$

$= 40$

Answer the following.

16. $(k + 1), 3k, (4k + 2)$

Common difference

$$3k - (k + 1) = (4k + 2) - 3k$$

$$2k - 1 = k + 2$$

$$k = 3$$

17. $\sqrt{\frac{\sec A - \tan A}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}}$

$= \sqrt{\dfrac{(\sec A - \tan A)^2}{\sec^2 A - \tan^2 A}}$

$= \sqrt{\dfrac{(\sec A - \tan A)^2}{1}}$ $(\sec^2 A - \tan^2 A = 1)$

$= \sec A - \tan A$

18. $= [(\sin^2 \theta)^2 - (\cos^2 \theta)^2 + 1] \operatorname{cosec}^2 \theta$

$= [(\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta) + 1] \operatorname{cosec}^2 \theta$

$= [\sin^2\theta - \cos^2\theta + 1] \operatorname{cosec}^2\theta$

$= [\sin^2\theta + (1 - \cos^2\theta)] \operatorname{cosec}^2\theta$

$= [\sin^2\theta + \sin^2\theta] \operatorname{cosec}^2\theta$

$= 2\sin^2\theta \times \dfrac{1}{\sin^2\theta} = 2$

19. Area of triangle formed by three points is zero if they are collinear

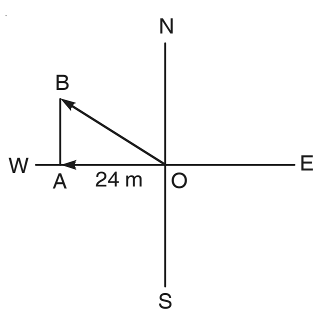
$\Rightarrow \dfrac{1}{2}[2(k - 7) + 5(7 - 3) + 6(3 - k)] = 0$

$\Rightarrow 2k - 14 + 20 + 18 - 6k = 0$

$-4k + 24 = 0$

$k = 6$

20.



$OB^2 = (24)^2 + (10)^2$

$= 576 + 100$

$OB^2 = 676 = (26)^2$

$OB = 26$

Section 'B'

21. Let the point p lie on x-axis

$\therefore$ P $(x, 0)$ is equidistant from point A (2, –5) and B (–2, 9)

$\therefore$ $PA^2 = PB^2$

or, $(2 - x)^2 + (5 - 0)^2 = (-2 - x)^2 + (9 - 0)^2$ **[1]**

$\Rightarrow 4 - 4x + x^2 + 25 = 4 + 4x + x^2 + 81$

$\Rightarrow -8x = 56$

$\Rightarrow x = -7$ **[1]**

$\therefore$ The point is (–7, 0).

22. Here, $a_1 = -1, a_2 = -5$ and $d = 4$

$S_n = \dfrac{n}{2}[2a + (n-1)d]$ **[1]**

$\therefore$ $S_{16} = \dfrac{16}{2}[2 \times (-1) + (16 - 1)(-4)]$

$S_{16} = 8[-2 - 60] = 8(-62)$

$\therefore$ $S_{16} = -496$ **[1]**

23. Here, $a_1 = 2, b_1 = 1, c_1 = -3$

and $a_2 = 4, b_2 = 2, c_2 = -6$

If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ then the lines are coincident

clearly $\dfrac{2}{4} = \dfrac{1}{2} = \dfrac{3}{6}$ **[1]**

Hence lines are coincident. **[1]**

24. $240 = 228 \times 1 + 12$

$228 = 12 \times 19 + 0$

Hence, HCF of 240 and 228 = 12. **[2]**

25. No. of cards = 20

Multiples of 5 from 11 to 30 are 15, 20, 25 and 30 so, number of favourable outcomes = 4 **[1]**

$\therefore$ Required probability $= \dfrac{4}{20} = \dfrac{1}{5}$ **[1]**

26. When two dices are thrown all possible outcomes = 6 × 6 = 36

If sum of both faces should be 10, they are {(4, 6), (6, 4), (5, 5)}

$\therefore$ No. of fovourable outcomes = 3 **[1]**

$\therefore$ Requred probability $= \dfrac{3}{36} = \dfrac{1}{12}$ **[1]**

Section 'C'

27. $x^2 - 2\sqrt{2x} = 0$

$x(x - 2\sqrt{2}) = 0$

Zeroes are 0 and $2\sqrt{2}$. **[1]**

Sum of zeroes = $2\sqrt{2} = -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$

and product of zeroes = $\dfrac{2\sqrt{2}}{1}$ **[1]**

$$0 = \frac{\text{constant term}}{\text{coefficient of } x^2} = 0.$$ **[1]**

28.

$$92 = 2^2 \times 23$$
$$510 = 2 \times 3 \times 5 \times 17$$
$$\text{HCF }(510, 92) = 2$$ **[1]**
$$\text{LCM }(510, 92) = 2^2 \times 23 \times 3 \times 5 \times 17$$
$$= 23460$$ **[1]**

HCF (510, 92) × LCM (510, 92)

$$= 2 \times 23460 = 46920$$

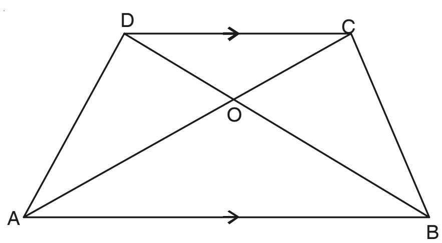
Product of two number

$$= 510 \times 92 = 46920$$

Hence, HCF × LCM

= Product of two numbers. **[1]**

29.



[1]

$$\Delta AOB \sim \Delta COD$$

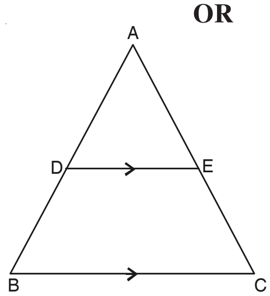
(AA similarity)

$$\frac{ar(\Delta COD)}{ar(\Delta AOB)} = \frac{CD^2}{AB^2}$$

$$= \frac{CD^2}{(3CD)^2} = \frac{CD^2}{9CD^2} = \frac{1}{9}$$ **[1]**

rotio = 1 : 9 **[1]**

OR



[1]

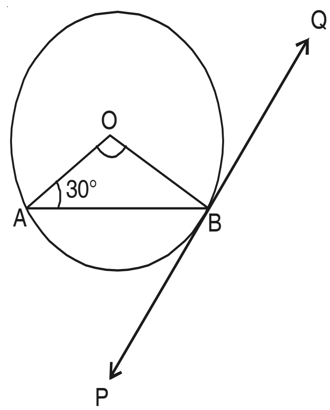
Given, DE ∥ BC

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{(By BPT)}$$

or, $$\frac{x+2}{3x+16} = \frac{x}{3x+5}$$ **[1]**

$$(x + 2)(3x + 5) = x(3x + 16)$$
$$3x^2 + 5x + 6x + 10 = 3x^2 + 16x$$
$$11x + 10 = 16x$$
$$16x - 11x = 10$$
$$5x = 10$$
$$x = 2$$ **[1]**

30.



[1]

Since the tangent is perpendicular to the end point of raduis,

$\therefore$ $\angle OBP = 90°$

$\angle OAB = \angle OBA$ $(\because OA = OB)$

$\angle OBA = 30°$ **[1]**

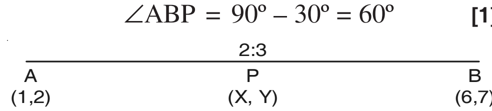
$\therefore$ $\angle AOB = 180° - (30° + 30°)$

$\angle AOB = 120°$

$\angle ABP = \angle OBP - \angle OBA$

$\angle ABP = 90° - 30° = 60°$ **[1]**

31.



$$AP = \frac{2}{5}AB$$

or, AP : PB = 2 : 3 **[1]**

$$x = \frac{2\times 6 + 3\times 1}{2+3}$$ **[½]**

and $$y = \frac{2\times 7 + 3\times 2}{2+3}$$ **[½]**

$\therefore$ $x = \dfrac{12+3}{5} = 3;$

$$y = \frac{14+6}{5} = 4$$

P (x, y) = (3, 4) **[1]**

OR

Since the points are collinear

$\therefore$ The area of triangle = 0 **[1]**

$\therefore$ Area of triangle

$$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$ **[½]**

$$\text{or, } \frac{1}{2}\left[5(4-y)+(-3)(y-2)+x(2-4)\right] = 0$$ **[½]**

$$\frac{1}{2}\left[20-5y-3y+6+(-2x)\right] = 0$$

$$\frac{1}{2}\left[-2x-8y+26\right] = 0$$

$$x + 4y - 13 = 0$$ **[1]**

Hence Proved.

32.

Class	x_i (class marks)	f_i	f_ix_i	*c.f.*
0–10	5	8	40	8
10–20	15	16	240	24
20–30	25	36	900	60
30–40	35	34	1190	94
40–50	45	6	270	100
		$\sum f_i = 100$	$\sum f_ix_i = 2640$	

[1]

$$\text{Mean} = \frac{\sum f_ix_i}{\sum f_i} = \frac{2640}{100} = 26.4$$

$$\text{Median} = \frac{N}{2}\text{th term}$$

$$= \frac{100}{2} = 50\text{th term}$$

Median Class = 20–30 **[½]**

$$\text{Median} = l_1 + \frac{\frac{N}{2} - cf}{f} \times h$$ **[½]**

$$\text{Median} = 20 + \frac{50-24}{36} \times 10$$

Median = 20 + 7.22 = 27.22 **[1]**

OR

Here, $r + h = 37$

and $2\pi r(r+h) = 1628$

$2\pi r \times 37 = 1628$ **[1]**

$$2\pi r = \frac{1620}{37}$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = \frac{1620}{37}$$

$$\Rightarrow \quad r = \frac{1628}{37} \times \frac{7}{2 \times 22}$$

$\Rightarrow$ $r = 7$ cm **[½]**

and $h = 30$ cm **[½]**

Hence volume of cylinder

$$= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30$$

Volume of cylinder = 4620 cm^3. **[1]**

33. Let the radius of sphere = R cm

Volume of sphere = Volume of cone

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$ **[1]**

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times 2 \times 2 \times 8$$

$$R^3 = \frac{2 \times 2 \times 8}{4}$$ **[1]**

$$R^3 = 8$$

$R = 2$ cm **[1]**

34. According to the question,

$\sin 3\theta = \cos(\theta - 6^\circ)$ **[1]**

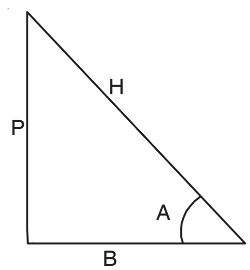
$\cos(90^\circ - 3\theta) = \cos(\theta - 6^\circ)$

$90^\circ - 3\theta = \theta - 6^\circ$ **[1]**

$4\theta = 90^\circ + 6^\circ = 96^\circ$

$$\theta = \frac{96^\circ}{4} = 24^\circ$$ **[1]**

OR



[1]

Let $\tan A = \frac{P}{B}$, $\sec A = \frac{H}{B}$

$$H^2 = P^2 + B^2$$ **[1]**

$$\text{LHS} = 1 + \tan^2 A$$

$$= 1+\left(\frac{P}{B}\right)^2 = 1+\frac{P^2}{B^2}$$

$$= \frac{B^2+P^2}{B^2} = \frac{H^2}{B^2} = \left(\frac{H}{B}\right)^2 = \sec^2 A = \text{RHS}$$

Hence Proved.

Section 'D'

35. Let the speed of the car I from A be x km/hr.
Speed of the car II from B be y km/hr

Same direction :

Distance covered by car I = 150 + (distance covered by car II)

$\Rightarrow \quad 15x = 150 + 15y$

$\Rightarrow \quad 15x - 15y = 150$

$\Rightarrow \quad x - y = 10$ ...(*i*) **[1]**

Opposite Direction :

Distance covered by car I + distance covered by car II

$= 150$ km

$x + y = 150$ ...(*ii*) **[1]**

Adding eq(*i*) and (*ii*),

$2x = 160$

$\therefore \quad x = 80$

Substituting $x = 80$ in eq (i),

$y = 70$

$\therefore$ Speed of the car I from A = 80 km/hr **[1]**

and speed of the car II from B = 70 km/hr **[1]**

OR

$x = -4$ is the root of the equation

$$x^2 + 2x + 4p = 0$$ **[1]**

$$(-4)^2 + 2(-4) + 4p = 0$$

$$16 - 8 + 4p = 0$$

$$p = -2$$ **[1]**

Equation $x^2 - 2(1 + 3k)x + 7(3 + 2k) = 0$ has equal roots

$$\therefore 4(1 + 3k)^2 - 28(3 + 2k) = 0$$

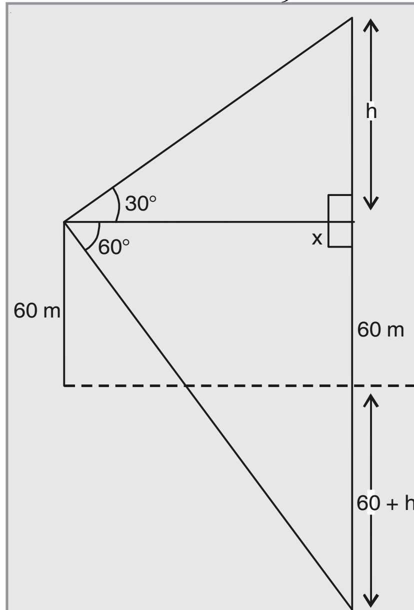
$$9k^2 - 8k - 20 = 0$$ **[1]**

$$(9k + 10)(k - 2) = 0$$

$$k = \frac{-10}{9}, 2$$

Hence, the value of $k = \frac{-10}{9}, 2$. **[1]**

36.



$$\frac{h}{x} = \tan 30^\circ \Rightarrow \frac{h}{x} = \frac{1}{\sqrt{3}}$$

$$x = h\sqrt{3}$$ ...(*i*)

and $\frac{h+60+60}{x} = \tan 60^\circ$

$$\Rightarrow \frac{h+120}{x} = \sqrt{3}$$

$$\Rightarrow h + 120 = x\sqrt{3}$$ ...(*ii*)

From (*i*) & (*ii*), we get

$$h + 120 = \sqrt{3}h \times \sqrt{3}$$

$$h + 120 = 3h$$

$$h = \frac{120}{2} = 60 \text{ m}$$

Hence height of a cloud from surface of water
$= 60 + 60 = 120$ m

[4] (CBSE Marking Scheme, 2017)

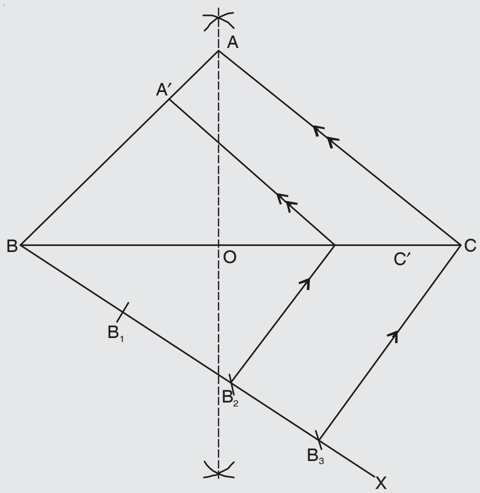
37. Steps of construction :

(i) Draw a line seqment BC = 8 cm

(ii) Draw perpendicular bisector of BC which intersects BC at O.

(iii) Mark A on bisector such that AO = 4 cm

(iv) Join A to B and A to C



[2]

(v) Draw an acute angle CBX at B fo BC, down word.

(vi) Mark B_1, B_2, B_3 on BX, such that $BB_1 = B_1B_2 = B_2B_3$

(vii) Join B_3 to C.

(viii) Draw $B_2C' \parallel B_3C$ to meet BC at C′

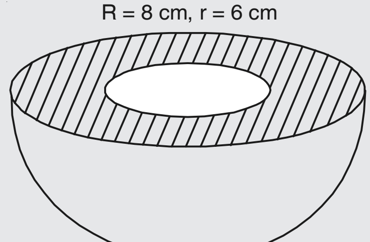
(ix) From C′ draw C′A′ ∥ CA, to meet AB at A′.

Hence A′BC′ is the required triangle.

[2 + 2] (CBSE Marking Scheme, 2017)

38.

R = 8 cm, r = 6 cm



Surface area $= 2\pi R^2 + 2\pi r^2 + \pi (R^2 - r^2)$

$= \pi [8^2 \times 2 + 6^2 \times 2 + (8^2 - 6^2)]$

$= \pi [64 \times 2 + 36 \times 2 + (64 - 36)]$

$= \pi [128 + 72 + 28]$

$= 228 \times 3.14$

$= 715.92 \text{ cm}^2$

$\therefore$ Total cost $= 715.92 \times 5 = ₹\ 3579.60$

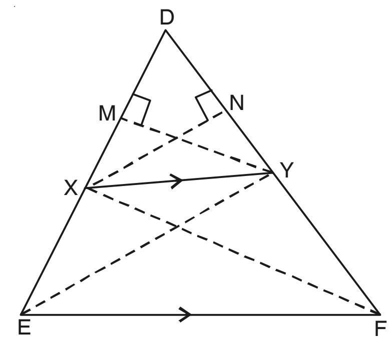
[4] (CBSE Marking Scheme, 2012)

39. Given : DEF is a triangle in which XY ∥ EF

To Prove : $\frac{DX}{XE} = \frac{DY}{YF}$

Construction : Draw XN ⊥ DY and YM ⊥ DX, Join EY and FX.

Proof : In ΔDEF,



area of $\Delta DXY = \frac{1}{2} \times DX \times YM$ ...(i)

area of $\Delta XYE = \frac{1}{2} \times XE \times YM$ ...(ii)

Dividing eq (i) by eq (ii)

$$\frac{\text{area of } \Delta DXY}{\text{area of } \Delta XYE} = \frac{\left(\frac{1}{2}\right) \times (DX) \times (YM)}{\frac{1}{2} \times (XE) \times (YM)}$$ **[1]**

$$\frac{\text{area of } \Delta DXY}{\text{area of } \Delta XYE} = \frac{DX}{XE}$$ ...(iii)

area of $\Delta DXY = \left(\frac{1}{2}\right) \times (DY) \times (XN)$ ...(iv)

area of $\Delta XYF = \left(\frac{1}{2}\right) \times (YF) \times (XN)$ ...(v)

Dividing eq (iv) by eq (v),

$$\frac{\text{area of } \Delta DXY}{\text{area of } \Delta XYF} = \frac{\left(\frac{1}{2}\right) \times (DY) \times (XN)}{\left(\frac{1}{2}\right) \times (YF) \times (XN)}$$ **[1]**

$$\frac{\text{area of } \Delta DXY}{\text{area of } \Delta XYF} = \frac{DY}{YF} \quad ...(vi)$$ **[1]**

ΔXYE and ΔXYF lie on the same base and between same parallel lines XY and EF.

area ΔXYE = area ΔXYF ...(vii)

From eq (vii),

$$\frac{\text{area of } \Delta DXY}{\text{area of } \Delta XYE} = \frac{DY}{YF} \quad ...(viii)$$

On comparing eq (iv) and eq (viii)

$$\frac{DX}{XE} = \frac{DY}{YF}$$ **[1]**

Hence Proved.

OR

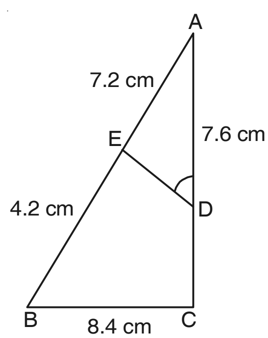
In ΔABC and ΔADE

∠ABC = ∠ADE [Given]

∠A = ∠A [Common]

ΔABC ~ ΔADE [AA Similarlity]

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$ **[1]**



[1]

$$\frac{AB}{AD} = \frac{BC}{DE}$$

$$\frac{AE+BE}{AD} = \frac{BC}{DE}$$

$$\frac{7.2+4.2}{7.6} = \frac{8.4}{DE}$$ **[1]**

$$\frac{11.4}{7.6} = \frac{8.4}{DE}$$

$$DE = \frac{8.4\times7.6}{11.4}$$

DE = 5.6 cm **[1]**

40.

Life Times	*c.f.*
Less than 1200	15
Less than 1400	75
Less than 1600	143
Less than 1800	229
Less than 2000	304
Less than 2200	365
Less than 2400	410

[2]

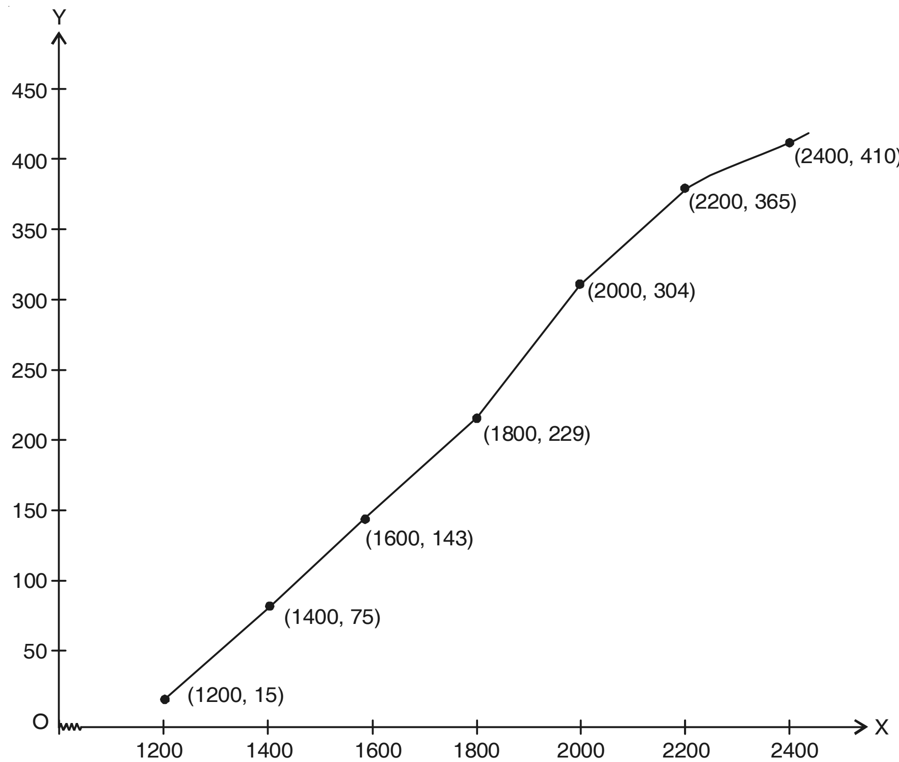
OR

Class	x_i (class marks)	f_i	f_ix_i
0–100	50	12	600
100–200	150	16	2400
200–300	250	6	1500
300–400	350	7	2450
400–500	450	9	4050
Total		$\Sigma f_i = 50$	$\Sigma f_ix_i = 11,000$

[2]

$$\text{Mean} = \frac{\sum f_ix_i}{\sum f_i} = \frac{11000}{50} = 220$$ **[1]**

Average daily income = ₹ 220. **[1]**


Y
450
400
350
300
250
200
150
100
50
O
1200
1400
1600
1800
2000
2200
2400
X
(1200, 15)
(1400, 75)
(1600, 143)
(1800, 229)
(2000, 304)
(2200, 365)
(2400, 410)

[1]

MATHEMATICS (STANDARD)

SOLUTIONS
SAMPLE QUESTION PAPER 10

CBSE Class X Examination

Section 'A'

1. (*b*) **2.** (*c*) **3.** (*c*) **4.** (*d*)
5. (*c*) **6.** (*b*) **7.** (*c*) **8.** (*c*)
9. (*b*) **10.** (*b*)

Fill in the blanks

11. 8 cm

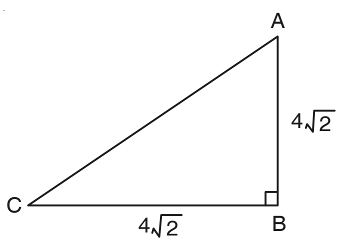
Explanation

$AC^2 = \left(4\sqrt{2}\right)^2 + \left(4\sqrt{2}\right)^2$

$AC^2 = 32 + 32$

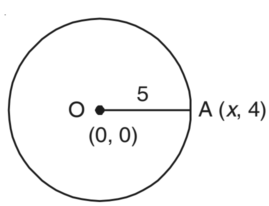
$AC^2 = 64$

$AC = 8$ cm



12. $x = \pm 3$

Explanation



Applying distance formula

$OA = \sqrt{(x-0)^2 + (4-0)^2}$

$5 = \sqrt{x^2 + 16}$

$25 = x^2 + 16$

$x^2 = 9$

$x = \pm 3$

13. $A = 45°$

Explanation

$\tan(A + B) = \sqrt{3} = \tan 60°$

$\therefore \quad A + B = 60°$ ... (i)

Similarly $\tan(A - B) = \dfrac{1}{\sqrt{3}} = \tan 30°$

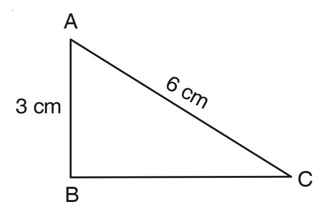
$\therefore \quad A - B = 30°$ ... (ii)

Solving (i) and (ii) $A = 45°$

14. $A = 60°$

Explanation

In ΔABC



$\cos A = \dfrac{AB}{AC}$

$\cos A = \dfrac{3}{6} = \dfrac{1}{2} = \cos 60°$

$\therefore \quad A = 60°$

15. 1

Explanation

Given $\cos 9\alpha = \sin \alpha$

$\Rightarrow \cos 9\alpha = \cos(90 - \alpha)$ As $(\cos(90 - \alpha) = \sin \alpha)$

$\Rightarrow \quad 9\alpha = 90 - \alpha$

$10\alpha = 90$

$\alpha = 9$

$\therefore \quad 5\alpha = 45$

Now $\tan 5\alpha = \tan 45° = 1$

Answer the following.

16. Let radius of circle bar

Circumference = Area of circle

$$2\pi r = \pi r^2$$

$\Rightarrow$ $r = 2$

$\therefore$ Diameter = 4

17. Volume of a ball with radius r

= Volume of 27 balls with radius r.

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = 27 \times \frac{4}{3}\pi r_1^3$$

$$\Rightarrow \quad \frac{r^3}{r_1^3} = \frac{27}{1}$$

$$\Rightarrow \quad \left(\frac{r}{r_1}\right)^3 = \left(\frac{3}{1}\right)^3$$

$\therefore$ $r : r_1 = 3 : 1$

18. Sum of zeroes $S = \frac{3}{5} + \left(\frac{-1}{2}\right) = \frac{1}{10}$

Product of zeroes = $P = \frac{3}{5} \times \left(-\frac{1}{2}\right) = -\frac{3}{10}$

$\therefore$ Required polynomial is $x^2 - 8x + P = 0$

$$\Rightarrow \quad x^2 - \frac{1}{10}x - \frac{3}{10} = 0$$

or $10x^2 - x - 3 = 0$

19. Given equation is $9x^2 + 8kx + 16 = 0$

Here $a = 9$, $b = 8k$ and $c = 16$

For equal roots $b^2 - 4ac = 0$

$(8k)^2 - 4 \times 9 \times 16 = 0$

$64k^2 - 64 \times 9 = 0$

$64(k^2 - 9) = 0$

$k^2 = 9$

or $k = \pm 3$

20. $S_n = 3x^2 + 5x$

$S_1 = 3 + 5 = 8 = a_1$

$S_2 = 3(2)^2 + 5(2) = 22 = a_1 + a_2$

$\therefore$ $a_2 = 22 - 8 = 14$

Now, the A.P. is 8, 14, ... 164

Here $a = 8$ $d = 6$ and $a_n = 164$

$a_n = a + (n - 1)d$

$164 = 8 + (n - 1)6$

$$\frac{156}{6} = n - 1$$

$n = 27$

Section 'B'

21. No, because $6^n = (2 \times 3)^n = 2^n \times 3^n$, so the only primes in the factorisation of 6^n are 2, 3 and not 5. **[1]**

Hence, it canot end with the digit 5. **[1]**

22. $ax + 3y = 5$

$a_1 = a$, $b_1 = 3$, $c_1 = 5$

$4x + 3ay = 10$

$a_2 = 4$, $b_2 = 3a$, $c_2 = 10$

For infinitely many solutions $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{a}{4} = \frac{3}{3a} = \frac{5}{10}$$ **[1]**

$$\Rightarrow \quad \frac{a}{4} = \frac{5}{10}$$

$\Rightarrow$ $a \times 10 = 5 \times 4$

$$\Rightarrow \quad a = \frac{20}{10}$$

$\Rightarrow$ $a = 2$ **[1]**

23.

$a + 3d = 0 \Rightarrow a = -3d$

$a_{25} = a + 24d = 21d$ **[1]**

$3a_{11} = 3(a + 10d)$

$= 3(7d) = 21d$ **[1]**

[CBSE Marking Scheme, 2016]

CBSE Topper's Answer, 2016

We have,

$a_4 = 0$

$a + 3d = 0$ $[a + (n-1)d = a_n]$

$3d = -a$

or $-3d = a$ — ①

Now,

$a_{25} = a + 24d$ $[a+(n-1)d = a_n]$

$-3d + 24d$ (Putting value of "a" from eq ①)

$= 21d$ — ②

$a_{11} = a + 10d$

$-3d + 10d$

$= 7d$ — ③ $(a = -3d)$

From eq ② & eq ③

$[a_{25} = 3a_{11}]$

Hence Proved.

24. Let the coordinates of points P and Q be (0, b) and (a, 0) resp.

$\therefore \quad \frac{a}{2} = 2 \Rightarrow a = 4$ **[½]**

$\frac{b}{2} = -5 \Rightarrow b = -10$ **[½]**

$\therefore$ P (0, –10) and Q (4, 0) **[1]**

[CBSE Marking Scheme, 2017]

25. (i) Probability of getting a doublet when two different dice are tossed. then

Total events $n(S) = 6 \times 6 = 36$

Favourable events i.e., to getting doublets

E = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$\therefore \quad n(E) = 6$

Probability of getting a doublet

$$= \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$\therefore$ Probability of getting a doublet $= \frac{1}{6}$ **[1]**

(ii) Favourable events of getting a sum 10, of the numbers on the two dice

E = {(4, 6), (5, 5), (6, 4)}

$\therefore \quad n(E) = 3$

Probability of getting a sum 10, of the numbers on the two dice

$$= \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$ **[1]**

26. No. of all possible outcomes = $6^2 = 36$

No. of favearable outcomes = 26 **[1]**

(4, 2) (4, 3) (4, 5) (5, 1) (5, 2) (5, 3) (3, 5) (6, 1) (6, 2) (1, 1) (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (2, 5) (4, 1)

$\therefore$ P (Product appears in less than 18)

$$= \frac{26}{36} = \frac{13}{18}$$ **[1]**

Section 'C'

27. Let us assume, to contrary that $4 - 3\sqrt{2}$ is a rational number.

$\therefore \quad 4 - 3\sqrt{2} = \frac{p}{q}$, $q \neq 0$ and $p, q \in Z$ **[1]**

$\Rightarrow \quad -3\sqrt{2} = \frac{p}{q} - 4$

$\Rightarrow \quad -3\sqrt{2} = \frac{p - 4q}{q}$

$\Rightarrow \quad \sqrt{2} = \frac{p - 4q}{-3q}$

$\Rightarrow \quad \sqrt{2} = \frac{4q - p}{3q} = \frac{\text{Integer}}{\text{Integer}}$ **[1]**

$\Rightarrow \quad \sqrt{2}$ = a rational number

But this contradicts the fact that $\sqrt{2}$ is irrational.

Hence, $4 - 3\sqrt{2}$ is an irrational number. **[1]**

28. Put $y = \frac{2x}{x-5}$ in the given equation

$$\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$$

Then, $\quad y^2 + 5y - 24 = 0$ **[1]**

where $y = \frac{2x}{x-5}$

$\Rightarrow y^2 + (8y - 3y) - 24 = 0$

$\Rightarrow (y^2 + 8y) - 3y - 24 = 0$

$\Rightarrow y(y + 8) - 3(y + 8) = 0$

$\Rightarrow (y - 3)(y + 8) = 0$

$\Rightarrow$ Either $y - 3 = 0$ or $y + 8 = 0$

$\Rightarrow$ Either $y = 3$ or $y = -8$ **[1]**

When $y = 3$, then

$$\frac{2x}{x-5} = 3$$

$\Rightarrow \quad 2x = 3x - 15$

$\Rightarrow \quad 3x - 2x = 15$

$\Rightarrow \quad x = 15$

When $y = -8$, then

$$\frac{2x}{x-5} = -8$$

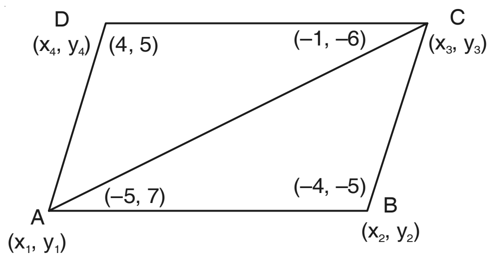
$\Rightarrow \quad 2x = -8x + 40$

$\Rightarrow \quad 10x = 40$

$\Rightarrow \quad x = 4$

Hence, the values of x are 4 and 15. **[1]**

29.



ar ΔABC

$$= \frac{1}{2}\left|\left[x_1(y_2 - y_3) + x_2(y_2 - y_3) + x_3(y_1 - y_2)\right]\right|$$

$$= \frac{1}{2}\left[-5(-5+6) - 4(-6-7) - 1(7+5)\right]$$

$$= \frac{1}{2}\left|(-5+52-12)\right|$$

ar ΔABC $= \frac{1}{2}|35| = \frac{35}{2}$ unit^2 **[1]**

ar ΔACD

$$= \frac{1}{2}\left|\left[x_1(y_3 - y_4) + x_3(y_4 - y_1) + x_4(y_1 - y_3)\right]\right|$$

$$= \frac{1}{2}\left|\left[-5(-6-5) + (-1)(5-7) + 4(7+6)\right]\right|$$

ar ΔACD

$$= \frac{1}{2}\left|(55+2+52)\right| = \frac{109}{2} \text{ unit}^2$$ **[1]**

ar of quadrilateral ABCD = ar ΔABC + ar ΔACD

$$= \frac{35}{2} + \frac{109}{2}$$

$$= \frac{144}{2} = 72 \text{ unit}^2$$

$\therefore$ ar of quadrilateral ABCD = 72 unit^2 **[1]**

OR

Let P (5, –2), Q (6, 4) and R (7, –2) be the given points.

$\because$ Distance between (x_1, y_1) and (x_2, y_2)

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

[by distance formula]

$$\therefore PQ = \sqrt{(6-5)^2 + (4+2)^2}$$

$$= \sqrt{(1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$ **[1]**

$$QR = \sqrt{(7-6)^2 + (-2-4)^2}$$

$$= \sqrt{(1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$ **[1]**

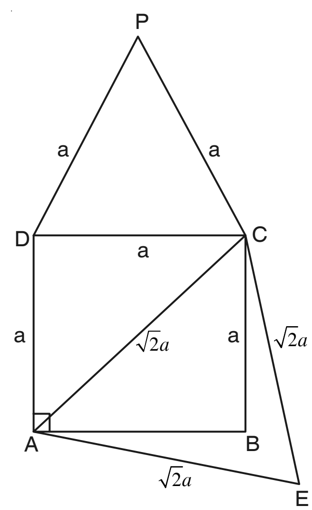
and $PR = \sqrt{(7-5)^2 + (-2+2)^2}$

$$= \sqrt{2^2 + 0} = 2 \text{ units}$$ **[1]**

$\because$ PQ = QR $\neq$ PR

$\therefore$ ΔPQR is an isosceles triangle.

30.



[1]

As ΔACE and ΔDCP are equilateral triangles

$\therefore$ ΔACE ~ ΔDCP (AAA)

$$\frac{ar\Delta ACE}{ar\Delta DCP} = \left(\frac{AC}{DC}\right)^2$$ **[1]**

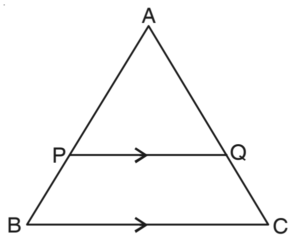
$$= \left(\frac{\sqrt{2}a}{a}\right)^2$$

$$= \frac{2}{1}$$

$\therefore$ ΔACE = 2 ar ΔDCP **[1]**

OR

Here, Since PQ ∥ BC and PQ divides ΔABC into two equal parts, So ΔAPQ ~ ΔABC



$\therefore \quad \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$

or, $\frac{1}{2} = \frac{AP^2}{AB^2}$ **[1]**

or, $\frac{1}{\sqrt{2}} = \frac{AB - BP}{AB}$ $(\because AB = AP + BP)$

or, $\frac{AB - BP}{AB} = \frac{1}{\sqrt{2}}$

or, $1 - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$ **[1]**

or, $\frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\therefore \quad BP : AB = (\sqrt{2}-1) : \sqrt{2}$ **[1]**

31. Let the radius of circle be r cm. Then,

$OA = OT = r$ cm.

Since, PT is a tangent to circle at T and OT is a radius.

So, $OT \perp PT$

$\therefore \quad \angle OTP = 90^\circ$ **[1]**

In right angled ΔOTP,

$OP^2 = OT^2 + PT^2$ ...(*i*)

[by Pythagoras theorem]

$\Rightarrow (PA + OA)^2 = OT^2 + 6^2$

$\Rightarrow (3 + r)^2 = r^2 + 36$

[from Eq. (*i*) and PA = 3 cm, PT = 6 cm, (given)]

$\Rightarrow r^2 + 6r + 9 - r^2 - 36 = 0$ **[1]**

$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow \quad 6r - 27 = 0 \quad \Rightarrow \quad r = \frac{27}{6} = 4.5$

Hence, the radius of the circle is 4.5 cm **[1]**

32. To Prove $(\cot\theta - \operatorname{cosec}\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

LHS $= (\cot\theta - \operatorname{cosec}\theta)^2$

$= \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2 = \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$ **[1]**

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$ **[1]**

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$ **[1]**

= RHS **Hence Proved.**

OR

Given: $x \sin\theta = y \cos\theta$

or, $x = \frac{y\cos\theta}{\sin\theta}$ ...(*i*)

and $x \sin^3\theta + y \cos^3\theta = \sin\theta\cos\theta$ ...(*ii*)

Substituting x from eqn. (*i*) in eqn. (*ii*),

$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

or, $y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$

or, $y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$

or, $y = \sin\theta$ ...(*iii*) **[1]**

Substituting this value of y in eqn. (*iii*),

$x = \cos\theta$ ...(*iv*) **[1]**

∴ Squaring and adding eqn. (*iii*) and eqn. (*iv*), we get

$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$ **[1]**

Hence Proved.

33. Volume of liquid in the bowl

$= \frac{2}{3}\cdot\pi\cdot(18)^3\text{ cm}^3$

Volume, after wastage $= \frac{2\pi}{3}\cdot(18)^3\cdot\frac{90}{100}\text{ cm}^3$

Volume of liquid in 72 bottles

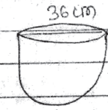
$= \pi(3)^2 .\, h\, .\, 72\text{ cm}^3$

$\Rightarrow \quad h = \frac{\frac{2}{3}\pi(18)^3\cdot\frac{9}{10}}{\pi(3)^2\cdot 72} = 5.4\text{ cm}$

[CBSE Marking Scheme, 2015]

CBSE Topper's Solution, 2016

13. Here, internal diameter of a hemispherical bowl = 36 cm



$\Rightarrow$ radius = R = 18 cm

A/Q

The liquid of the bowl is filled into 72 cylindrical bottles of diameter 6 cm i.e of radius 3 cm

Also, 10% liquid is wasted.

Now, vol.^{m} of the bowl $= \frac{2}{3} \times \pi \times (18)^3 = \frac{2 \times \pi \times 18 \times 18 \times 18}{3}$

$= (12 \times 324\, \pi)\ cm^3$

Liquid wasted $= \frac{10}{100} \times 12 \times 324\, \pi = \left(\frac{1944}{5}\pi\right) cm^3$

Now, liquid used to fill the bottles $= \left(3888\pi - \frac{1944\pi}{5}\right) cm^3$

$= \left(\frac{19440\pi - 1944\pi}{5}\right) cm^3$

$= \frac{17496\pi}{5}\ cm^3$

Now, height of each bottle $= \frac{17496\pi}{5 \times \pi \times 3 \times 3 \times 72}$

$= \frac{27}{5}$ cm

$= 5.4$ cm

OR

Slant height (l) = $\sqrt{(2.8)^2 + (2.1)^2}$ = 3.5 cm

$\therefore$ Area of canvas = $2 \times \frac{22}{7} \times (2.1) \times 4 + \frac{22}{7} \times 2.1 \times 3.5$

for one tent = 6.6 (8 + 3.5) = 6.6 × 11.5 m^2 **[1]**

$\therefore$ Area for 100 tents = 66 × 115 m^2

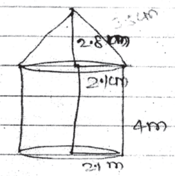
Cost of 100 tents = ₹ 66 × 115 × 100

50% Cost = 33 × 11500 = ₹ 379500 **[1]**

Values : Helping the flood victims. **[CBSE Marking Scheme, 2015] [1]**

CBSE Topper's Solution, 2015

Sec-C.



11. Here, Height of cylindrical part (H) = 4m

Base radius = $\frac{4.2}{2}$ = 2.1 m = r

Now, Height of conical part (h) = 2.8 m

Now, slant height of cone (l) = $\sqrt{h^2+r^2}$

$= \sqrt{(2.8)^2+(2.1)^2}$

$= \sqrt{\{7(0.4)\}^2+\{7(0.3)\}^2}$

$= \sqrt{49(0.16+0.09)} = 7\sqrt{0.25}$

$= 7 \times 0.5$

$= 3.5$ m

[redacted]

Req. area of canvas for making 1 tent

= C.S.A of cone + C.S.A of cylinder

$= \pi r l + 2\pi r H$

$= \pi r[l+2H] = \pi \times 2.1[3.5+2\times4]$

$= \frac{22}{7} \times \frac{21}{10}[3.5+8]$

$= \frac{33}{5}[11.5]$ m² $= \frac{33 \times 115}{5 \times 10} = \frac{33 \times 23}{10}$

Req. area of canvas for making 100 tents $= 100 \times \frac{33}{5} \times \frac{23}{10}$

$= \frac{100 \times 33 \times 23}{10}$

$= (33 \times 230)$ m²

∵ Rate of canvas = ₹ 100 / m²

∴ cost " " = ₹ 100 × 33 × 230

= ₹ (3300 × 230)

Since, the welfare association will contribute 50%.

∴ Its contribution $= ₹\frac{50}{100} \times 3300 \times 230$

= ₹ 16500 × 23

= ₹ 379500

The associations are very social, helpful, generous & kind.

34. Let assumed mean, a = 35 and given h = 10.

Class	x_i (Class Marks)	$u_i = \frac{x_i - a}{h}$	f_i	f_iu_i
0 – 10	5	–3	5	–15
10 – 20	15	–2	13	–26
20 – 30	25	–1	20	–20
30 – 40	35	0	15	0
40 – 50	45	1	7	7
50 – 60	55	2	5	10
	Total		$\Sigma f_i = 65$	$\Sigma f_iu_i = -44$

[2]

$\therefore$ Mean, $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 35 + \frac{-44}{65} \times 10$

$= 35 - 6.76 = 28.24$

Section 'D'

35. Discriminant $= b^2 - 4ac = 36 - 4 \times 5 \times (-2)$

$= 76 > 0$

So, the given equation has two distinct real roots

$5x^2 - 6x - 2 = 0$ **[1]**

Multiplying both sides by 5.

$(5x)^2 - 2 \times (5x) \times 3 = 10$

$\Rightarrow (5x)^2 - 2 \times (5x) \times 3 + 3^2 = 10 + 3^2$

$\Rightarrow (5x - 3)^2 = 19$ **[1]**

$\Rightarrow 5x - 3 = \pm\sqrt{19}$

$\Rightarrow x = \frac{3 \pm \sqrt{19}}{5}$ **[1]**

Verification:

$5\left(\frac{3+\sqrt{19}}{5}\right)^2 - 6\left(\frac{3+\sqrt{19}}{5}\right) - 2$

$= \frac{9+6\sqrt{19}+19}{5} - \frac{18+6\sqrt{19}}{5} - \frac{10}{5} = 0$

Similarly, $5\left(\frac{3-\sqrt{19}}{5}\right)^2 - 6\left(\frac{3-\sqrt{19}}{5}\right) - 2 = 0$

[1]

[CBSE Marking Scheme, 2018]

OR

Let the length of shorter side be x m.

$\therefore$ length of diagonal $= (x + 16)$ m **[1]**

and, lenght of longer side $= (x + 14)$ m **[1]**

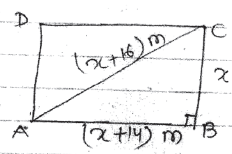
$\therefore x^2 + (x + 14)^2 = (x + 16)^2$ **[1]**

$\Rightarrow x^2 - 4x - 60 = 0 \Rightarrow x = 10$ m

$\therefore$ length of sides are 10m and 24m. **[1]**

CBSE Topper's Solution, 2015

Let ABCD be a rectangle.
Let the shorter side, BC = x cm
Then, AC = $(x+16)$ cm
And, AB = $(x+14)$ cm



Now, in right ΔABC, By Pythagoras theorem

$AC^2 = AB^2 + BC^2$

or, $(x+16)^2 = (x+14)^2 + x^2$

or, $x^2 + 256 + 32x = x^2 + 196 + 28x + x^2$

or, $2x^2 + 28x + 196 - x^2 - 32x - 256 = 0$

or, $x^2 - 4x - 60 = 0$, which is a Quad. eqn.

or, $x^2 - 10x + 6x - 60 = 0$

or, $x(x-10) + 6(x-10) = 0$

or, $(x-10)(x+6) = 0$

or, $x - 10 = 0$ | or, $x + 6 = 0$

or, $x = 10$ | or, $x = -6$ (invalid)

Now, BC = x = 10 m

AB = $x+14$ = $(10+14)$ m = 24 m

36. In ΔABC, $\angle B = 90°$, AD and CE are two medians.

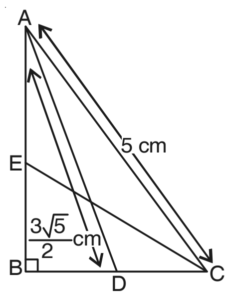
$\therefore$ $AC^2 = AB^2 + BC^2 = (5)^2 = 25$ **[1]**

(By Pythagoras theorem) ...(*i*)

In ΔABD, $AD^2 = AB^2 + BD^2$

or, $$\left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$$

or, $$\frac{45}{4} = AB^2 + \frac{BC^2}{4}$$ ...(*ii*)



[1]

In ΔEBC, $$CE^2 = BC^2 + \frac{AB^2}{4}$$ ...(*iii*)

Subtracting equation (*ii*) from equation (*i*),

$$\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$$

or, $$BC^2 = \frac{55}{3}$$ ...(*iv*) **[1]**

From eqn. (*ii*),

$$AB^2 + \frac{55}{12} = \frac{45}{4}$$

or, $$AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$$

From eqn. (*iii*), $$CE^2 = \frac{55}{3} + \frac{20}{3 \times 4} = \frac{240}{12} = 20$$

$$CE = \sqrt{20} = 2\sqrt{5} \text{ cm}$$ **[1]**

OR

Given : $\Delta ABC \sim \Delta PQR$

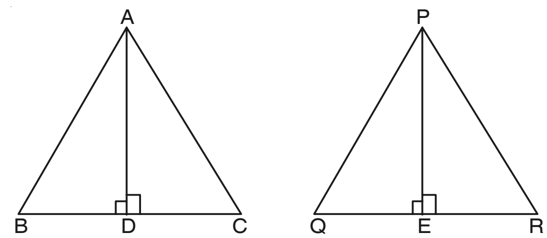
To Prove :

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$$

$$= \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$ **[1]**

Construction : Draw $AD \perp BC$ and $PE \perp QR$

Proof : $\Delta ABC \sim \Delta PQR$



$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

(Corresponding sides of similar triangles) ...(*i*)

In ΔADB and ΔPEQ,

$\angle B = \angle Q$ (Proved)

$\angle ADB = \angle PEQ$ [each 90°]

$\therefore$ $\Delta ADB \sim \Delta PEQ$ (AA similarity)

or, $$\frac{AD}{PE} = \frac{AB}{PQ}$$

(Corresponding sides of similar triangles) ...(*ii*)

From eq. (*i*) and eq. (*ii*),

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PE}$$ **[1]** ...(*iii*)

Now $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PE}$$

$$= \left(\frac{BC}{QR}\right) \times \left(\frac{AD}{PE}\right)$$

$$= \frac{BC}{QR} \times \frac{BC}{QR}$$

or, $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC^2}{QR^2}$ ...(*iv*) **[1]**

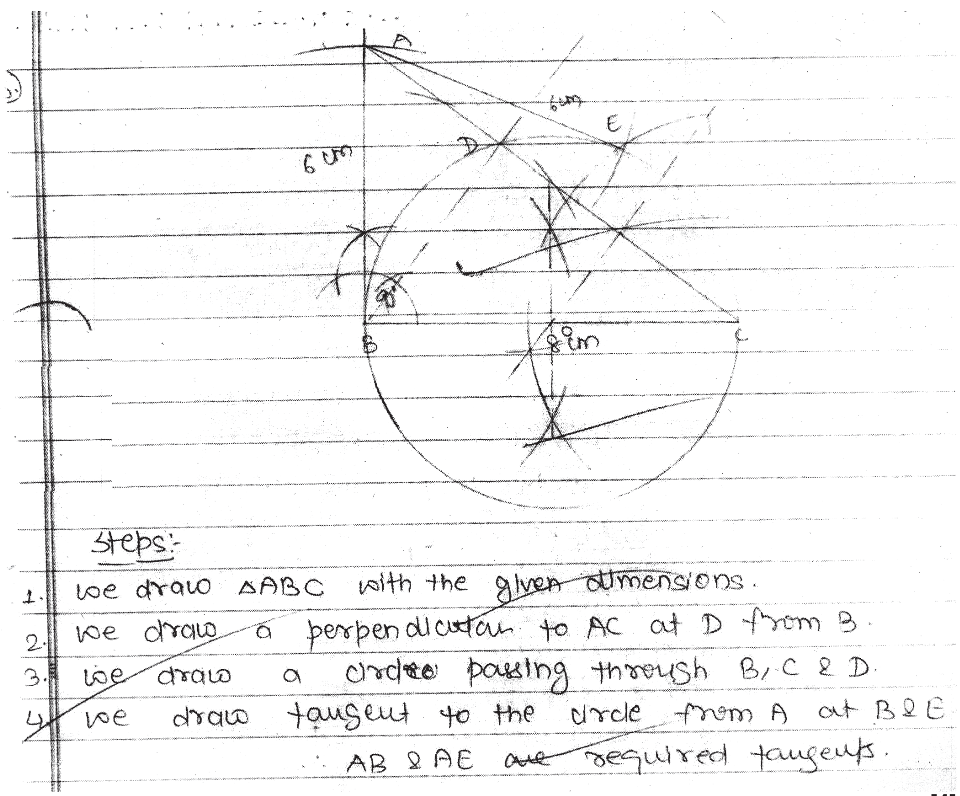
[frome eq. (*iii*)]

Frome eq (*iii*) and eq (*iv*),

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$$

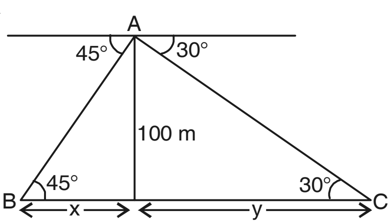
$$= \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \quad \textbf{[1]}$$

37. CBSE Topper's Solution, 2015



[4]

38.



[1]

$$\frac{100}{x} = \tan 45° = 1$$

$\Rightarrow$ $x = 100$ ...(*i*) **[1]**

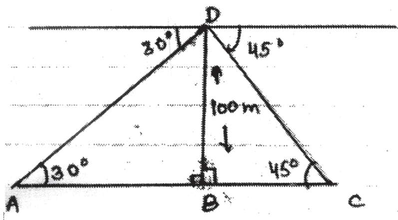
$$\frac{100}{y} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $y = 100\sqrt{3}$ ...(*ii*) **[1]**

Distance between the cars = $x + y$

$$= 100\left(\sqrt{3}+1\right) = 273.2 \text{ m}$$ **[1]**

CBSE Topper's Solution, 2017



To find : AC

Solution:

In $\triangle ABD$, $\angle DAB = 30°$

In $\triangle BDC$, $\angle BCD = 45°$.

also, $BD = 100$m.

In right $\triangle ABD$,

$$\tan 30° = \frac{DB}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{AB}$$

$$AB = 100\sqrt{3} = 100 \times 1.732 = 173.2 \text{ m}$$

In right $\triangle DBC$,

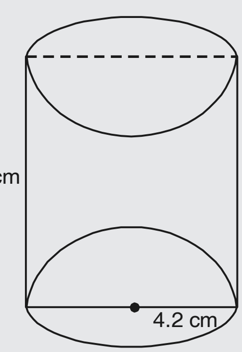
$$\tan 45° = \frac{DB}{BC}$$

$$1 = \frac{100}{BC} \Rightarrow BC = 100 \text{m}.$$

Now, $AC = AB + BC = 100 + 173.2 \text{ m} = 273.2 \text{ m}$ or $100(\sqrt{3}+1)$ m

39. Total Volume of cylinder =

$$\frac{22}{7}\times\frac{22}{10}\times\frac{42}{10}\times 10 \text{ cm}^3 = 554.40 \text{ cm}$$



[1]

Volume of metal scooped out =

$$\frac{4}{3}\times\frac{22}{7}\times\left(\frac{42}{10}\right)^3 = 310.46 \text{ cm}^3$$ **[1]**

$\therefore$ Volume of rest of cylinder $= 554.40 - 310.46 = 243.94 \text{ cm}^3$ **[1]**

If l is the length of wire, then

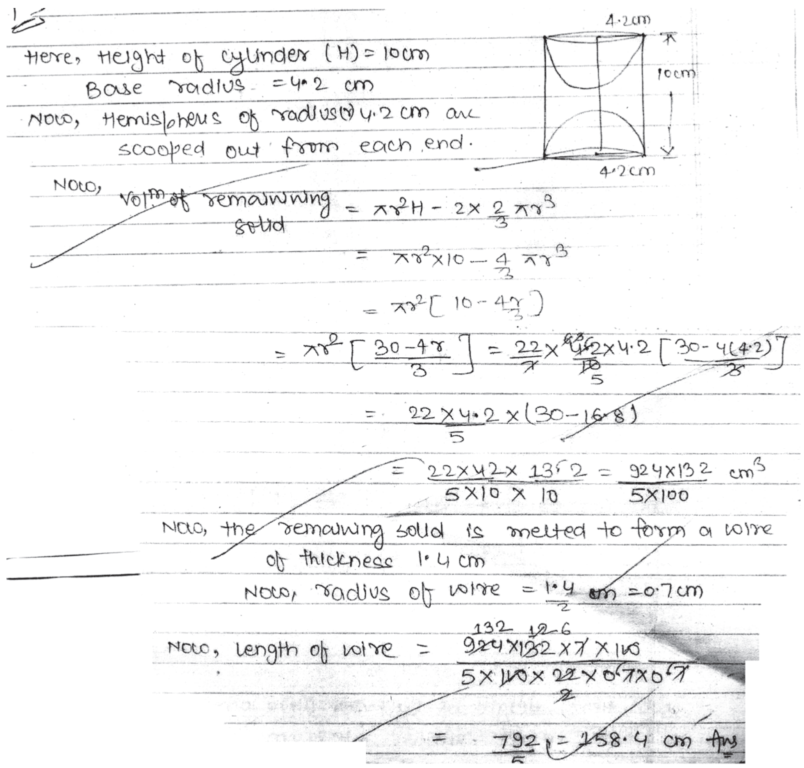
$$\frac{22}{7}\times\frac{7}{10}\times\frac{7}{10}\times l = \frac{24394}{100}$$

$$\Rightarrow \quad l = 158.4 \text{ cm}$$

[1]

[CBSE Marking Scheme, 2017]

CBSE Topper's Solution, 2015



40. (i) By Formula Method :

Classes	f	$c.f.$	
0–20	6	6	
20–40	8	14	
40–60	10	24	
60–80	12	36	⇒ Median Class
80–100	6	42	
100–120	5	47	
120–140	3	50	

[1]

$$\text{Median} = \frac{N}{2}\text{th term} = \frac{50}{2} = 25\text{th term}$$

Median class = 60 – 80

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h = 60 + \left(\frac{25-24}{12}\right) \times 20$$

$$= 60 + \frac{1}{12} \times 20 = 60 + \frac{5}{3} = \frac{185}{3} = 61.67$$

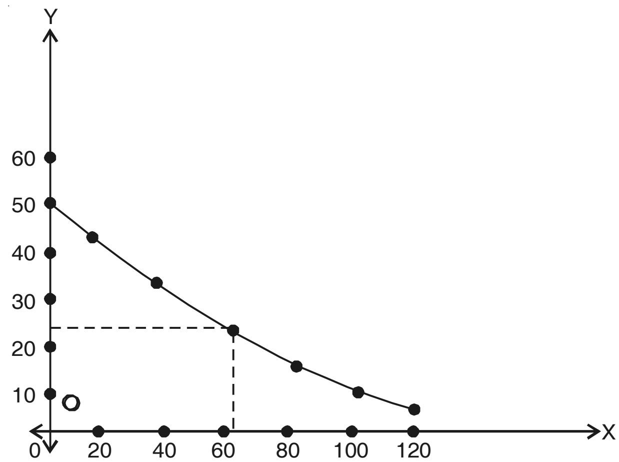
[½]

(ii)

Classes	*c.f.*
More than 0	50
More than 20	44
More than 40	36
More than 60	26
More than 80	14
More than 100	8
More than 120	3

[1]

To draw ogive we take the indeces : (0, 50), (20, 44), (40, 36), (60, 26), (80, 14), (100, 8), (120, 3)



[1]

From graph, $\frac{N}{2} = \frac{50}{2} = 25$

Median = 61.6 **[½]**

OR

Class	x_i (Class marks)	f_i	$f_i x_i$
0–30	15	12	180
30–60	45	21	945
60–90	75	x	$75x$
90–120	105	52	5460
120–150	135	y	$135y$
150–180	165	11	1815
	Total	$\Sigma f_i = x + y + 96 = 150$	$\Sigma f_i x_i = 8400 + 75x + 135y$

[2]

$96 + x + y = 150$ ...(*i*)

$x + y = 54$

$\therefore \quad x = \frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 91 = \frac{8400 + 75x + 135y}{150}$

$\Rightarrow \quad 13650 = 8,400 + 75x + 135y$

$\Rightarrow 75x + 135y = 5250$

$\Rightarrow \quad 5x + 9y = 350$ ...(*ii*) **[1]**

Solving eqns. (*i*) and (*ii*),

$x = 34$ and $y = 20$ **[1]**

MATHEMATICS (STANDARD)

SOLUTIONS SAMPLE QUESTION PAPER 11

CBSE Class X Examination

Section 'A'

1. (*d*)	**2.** (*b*)	**3.** (*a*)	**4.** (*d*)
5. (*d*)	**7.** (*b*)	**7.** (*d*)	**8.** (*b*)
9. (*c*)	**10.** (*c*)		

Fill in the blanks

11. $p = 4$

Explanation

$(2p + 1)$, 13, $(5p - 3)$ are in A.P.

Common difference $= (5p - 3) - 13$

$= 13 - (2p + 1)$

$\Rightarrow \quad 5p - 16 = 12 - 2p$

$7p = 28$

$p = 4$

12. 2139

Explanation

Here $a = 5$ and $d = 13 - 5 = 8$

$a_x = a + (n - 1)d$

$|8| = 5 + (n - 1)8$

$176 = 8(x - 1)$

$x - 1 = 22$

$x = 23$

$$\therefore \quad S_n = \frac{n}{2}[a + a_n]$$

$$= \frac{23}{2}[5+181]$$

$$= \frac{23}{2}\times 186 = 2139$$

13. $x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

Explanation

$$3x^2 - 2\sqrt{6}x + 2 = 0$$

$$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$3x^2 - \sqrt{3}\sqrt{2}x - \sqrt{3}\sqrt{2}x + 2 = 0$$

$$\sqrt{3}x[\sqrt{3}x - \sqrt{2}] - \sqrt{2}[\sqrt{3}x - \sqrt{2}] = 0$$

$$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$$

14. $y = 2/3$ and $-1/7$

Explanation

$$7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$$

$\Rightarrow \quad 21y^2 - 11y - 2 = 0$

$\Rightarrow 21y^2 - 14y + 3y - 2 = 0$

$7y(3y - 2) + 1(3y - 2) = 0$

$(3y - 2)(7y + 1) = 0$

$y = 2/3$ and $-1/7$

15. LCM = 338

Explanation

Given $N_1 = 26$

$N_2 = 169$

and H.C.F. = 13

We know that $N_1 \times N_2 = \text{HCF} \times \text{LCM}$

$\Rightarrow \quad 26 \times 169 = 13 \times \text{LCM}$

$\therefore \quad \text{LCM} = 338$

Answer the following.

16. Let outer radius of path be R and inner radius be r.

According to question:

$$\frac{2\pi R}{2\pi r} = \frac{23}{22}$$

$$\Rightarrow \quad \frac{R}{r} = \frac{23}{22} \quad \text{... (i)}$$

Also $R - r = 5$m

$$\Rightarrow \quad R = (5 + r)\text{m} \quad \text{... (ii)}$$

Solving (i) and (ii)

$$\frac{5+r}{r} = \frac{23}{22}$$

$$110 + 22r = 23r$$

$$\Rightarrow \quad r = 110 \text{ m}$$

$\therefore$ Diameter = 220 m

17. Given $Q = 45°$

$r = 7$ cm

Arc length $l = \frac{Q}{360} \times 2\pi r$

$$l = \frac{45}{360} \times 2 \times \frac{22}{7} \times 7$$

$= 5.5$ cm

18. Curved surface area $= 2\pi rh = 264$ ... (i)

Volume $= \pi r^2 h = 264$ ... (ii)

Dividing (i) ÷ ii $\frac{2\pi rh}{\pi r^2 h} = \frac{264}{924}$

$$\frac{2}{r} = \frac{2}{7}$$

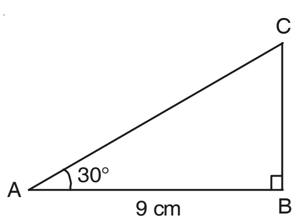
$\Rightarrow \quad r = 7 \quad \Rightarrow \quad$ Diameter = 14

from (i) $2 \times \frac{22}{7} \times 7 \times h = 264$

$h = 6$

$$\therefore \quad \frac{\text{Diameter}}{\text{height}} = \frac{14}{6} = \frac{7}{3}$$

19.



In ΔABC

$$\tan 30° = \frac{BC}{9}$$

$$\frac{1}{\sqrt{3}} = \frac{BC}{9} \Rightarrow BC = \frac{9}{\sqrt{3}}$$

$$BC = \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3\sqrt{3} \text{ cm}$$

20. $\sin A = \cos B$

$\sin A = \sin (90 - B)$

$\Rightarrow A = 90 - B$

$\Rightarrow A + B = 90°$

Section 'B'

21. $kx + y = k^2$ and $x + ky = 1$

$$\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1} \quad \textbf{[1]}$$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{1} = \frac{1}{k} = \frac{k^2}{1} = k^2 = 1$$

$$k = \pm 1 \quad \textbf{[1]}$$

22. Let us assume to the contrary, that $3\sqrt{7}$ is an rational.

Then, there exist co-prime positive integers p and q such that

$$3\sqrt{7} = \frac{p}{q} \quad \textbf{[1]}$$

$$\left[\begin{array}{l} \because p, 3 \text{ and } q \text{ are integers} \\ \therefore \frac{p}{3q} \text{ is a rational number} \end{array}\right]$$

$$\Rightarrow \quad \sqrt{7} = \frac{p}{3q}$$

$\Rightarrow \sqrt{7}$ is rational **[1]**

But this contradicts the fact that $\sqrt{7}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3\sqrt{7}$ is rational. So, we conclude that $3\sqrt{7}$ is an irrational number.

23. Here, first term $(a) = 6$

Common difference (d) of the given

$$\text{A.P.} = 13 - 6 = 7 \quad \textbf{[1]}$$

Let the given A.P. contains n terms, then

$$t_n = 216 \text{ (given)}$$

$\Rightarrow \quad a + (n-1)d = 216$

$\Rightarrow \quad 6 + (n-1)7 = 216$

$\Rightarrow \quad (n-1)7 = 210$

$\Rightarrow \quad n - 1 = 30$

$\Rightarrow \quad n = 30 + 1 = 31$

Thus, the given A.P. contains 31 terms. **[1]**

24. Let the point be A(3, 0), B(6, 4), C(–1, 3)

$AB = \sqrt{9+16} = 5,$

$BC = \sqrt{49+1} = 5\sqrt{2},$

$AC = \sqrt{16+9} = 5$

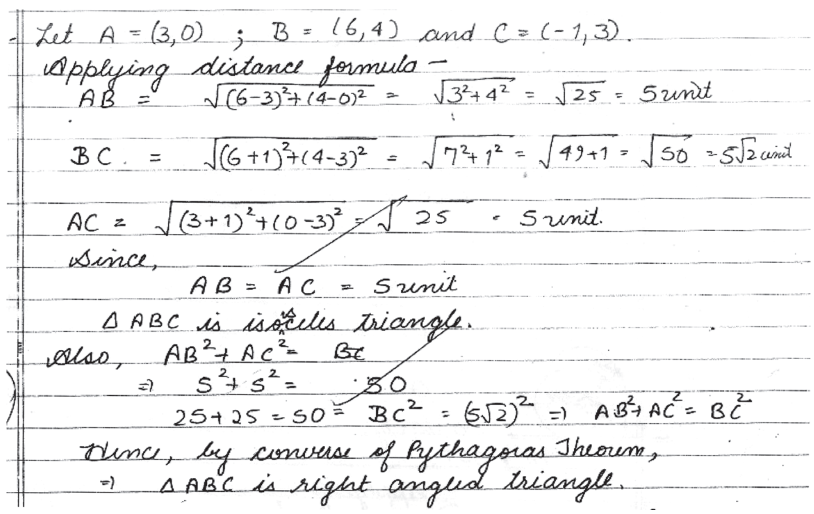
$AB = AC$

and $AB^2 + AC^2 = BC^2$

ΔABC isosceles right angled triangle.

[1½ + ½] [CBSE Marking Scheme, 2016]

CBSE Topper's Answer, 2016



Let A = (3,0) ; B = (6,4) and C = (−1,3).
Applying distance formula –
$AB = \sqrt{(6-3)^2+(4-0)^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5$ unit
$BC = \sqrt{(6+1)^2+(4-3)^2} = \sqrt{7^2+1^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$ unit
$AC = \sqrt{(3+1)^2+(0-3)^2} = \sqrt{25} = 5$ unit.
Since,
AB = AC = 5 unit
ΔABC is isosceles triangle.
Also, $AB^2 + AC^2 = BC$
$\Rightarrow 5^2 + 5^2 = 50$
$25 + 25 = 50 = BC^2 = (5\sqrt{2})^2 \Rightarrow AB^2 + AC^2 = BC^2$
Hence, by converse of Pythagoras Theorem,
$\Rightarrow \Delta ABC$ is right angled triangle.

25. **(i)** P(squre number) $= \dfrac{8}{113}$ **[1]**

(ii) P(multiple of 7) $= \dfrac{16}{113}$ **[1]**

[CBSE Marking Scheme, 2018]

26. No. of possible outcomes = 100

Perfect squares 4, 9, 16, 25, 36, 49, 64, 81, 100

No. of favourable outcomes = 9

(i) P(perfect square) $= \dfrac{9}{100}$ **[1]**

(ii) P(odd numbers not less than 70)

$$= \frac{16}{100} = \frac{4}{25} \quad \textbf{[1]}$$

Section 'C'

27. According to the statement of the question, we have

LCM of two numbers = 14 × HCF of two numbers

Also, $\text{LCM} + \text{HCF} = 600$ **[1]**

$\Rightarrow \quad 14 \times \text{HCF} + \text{HCF} = 600$

$\Rightarrow \quad 15\ \text{HCF} = 600$

$\Rightarrow \quad \text{HCF} = 40$ **[1]**

$\therefore \quad \text{LCM} = 14 \times 40 = 560$

Now, one number is 280

$\therefore \quad 280 \times \text{Other number} = 40 \times 560$

$\Rightarrow \quad \text{Other number} = \dfrac{40 \times 560}{280}$

$= 80$ **[1]**

28. Let the ten's and the units digit be y and x respectively.

So, the number is $10y + x$.

The number when digits are reversed is $10x + y$.

Now, $7(10y + x) = 4(10x + y) \Rightarrow 2y = x$ ...(*i*)

Also $x - y = 3$ ...(*ii*)

Solving (1) and (2), we get $y = 3$ and $x = 6$.

[3] [CBSE Marking Scheme, 2018]

29. $PA = PB$ or $(PA)^2 = (PB)^2$

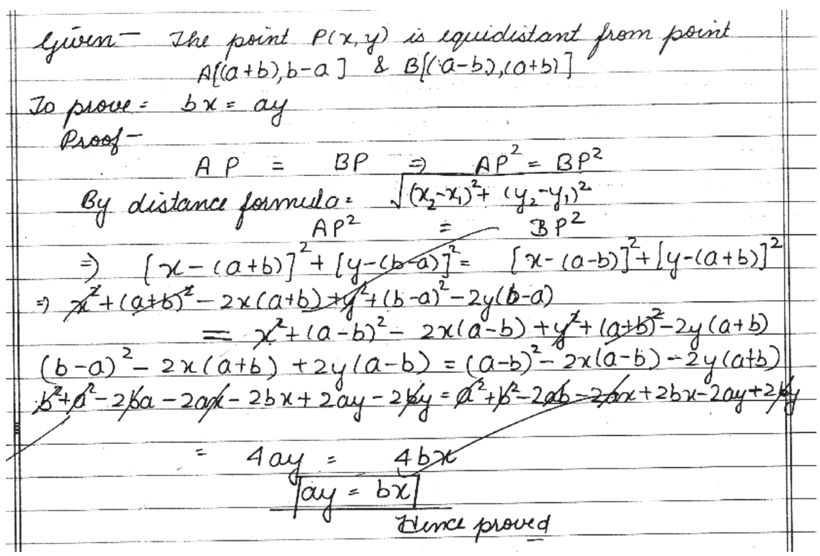
$$(a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$$

$$(a + b)^2 + x^2 - 2ax - 2bx + (b - a)^2 + y^2 - 2by + 2ay = (a - b)^2 + x^2 - 2ax + 2bx + (a + b)^2 + y^2 - 2ay - 2by$$

$\Rightarrow 4ay = 4bx$ or $bx = ay$

[3] [CBSE Marking Scheme, 2015]

CBSE Topper's Solution 2015



Given – The point P(x, y) is equidistant from point A[(a+b), b−a] & B[(a−b), (a+b)]

To prove = bx = ay

Proof –

$AP = BP \Rightarrow AP^2 = BP^2$

By distance formula = $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$AP^2 = BP^2$

$\Rightarrow [x-(a+b)]^2 + [y-(b-a)]^2 = [x-(a-b)]^2 + [y-(a+b)]^2$

$\Rightarrow x^2 + (a+b)^2 - 2x(a+b) + y^2 + (b-a)^2 - 2y(b-a)$

$= x^2 + (a-b)^2 - 2x(a-b) + y^2 + (a+b)^2 - 2y(a+b)$

$(b-a)^2 - 2x(a+b) + 2y(a-b) = (a-b)^2 - 2x(a-b) - 2y(a+b)$

$b^2 + a^2 - 2ba - 2ax - 2bx + 2ay - 2by = a^2 + b^2 - 2ab - 2ax + 2bx - 2ay + 2by$

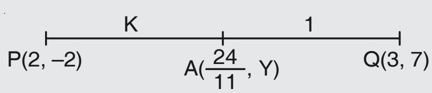
$= 4ay = 4bx$

$ay = bx$

Hence proved

OR

Let PA : AQ = k : 1



[1]

$\therefore \quad \dfrac{2+3k}{k+1} = \dfrac{24}{11}$ **[½]**

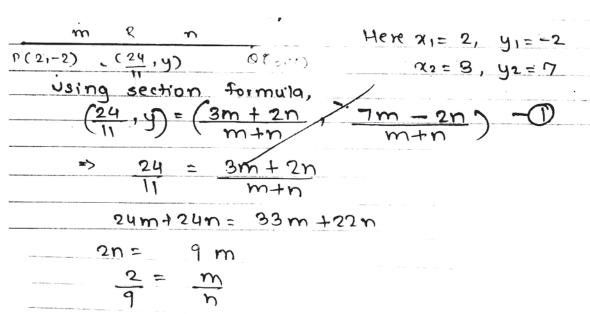
$\Rightarrow \quad k = \dfrac{2}{9}$

Hence the ratio is 2 : 9. **[½]**

Therefore $\quad y = \dfrac{-18+14}{11} = \dfrac{-4}{11}$ **[1]**

[CBSE Marking Scheme, 2017]

CBSE Topper's Solution 2017



m ℓ n

P(2, −2) (24/11, y) Q(3, 7)

Here $x_1 = 2$, $y_1 = -2$

$x_2 = 3$, $y_2 = 7$

Using section formula,

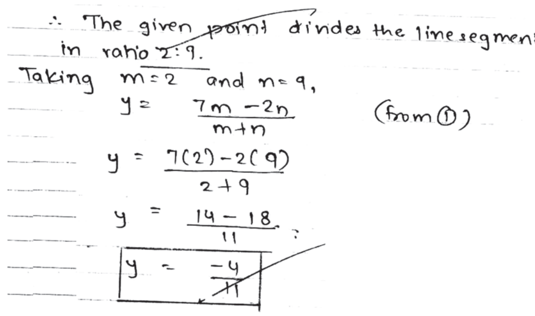
$\left(\frac{24}{11}, y\right) = \left(\frac{3m+2n}{m+n}, \frac{7m-2n}{m+n}\right)$ —①

$\Rightarrow \frac{24}{11} = \frac{3m+2n}{m+n}$

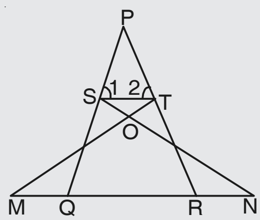
$24m + 24n = 33m + 22n$

$2n = 9m$

$\frac{2}{9} = \frac{m}{n}$


∴ The given ~~point~~ divides the line segment in ratio 2 : 9.
Taking m = 2 and n = 9,
$y = \frac{7m - 2n}{m+n}$ (from ①)
$y = \frac{7(2) - 2(9)}{2+9}$
$y = \frac{14 - 18}{11}$
$y = \frac{-4}{11}$

30. $\angle SQN = \angle TRM$ (CPCT as $\Delta NSQ \cong \Delta MTR$)



Since, $\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$

(Angle sum property)

$\Rightarrow \quad \angle 1 + \angle 2 = \angle PQR + \angle PRQ$

$\Rightarrow \quad 2\angle 1 = 2\angle PQR$

(as $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$)

$\angle 1 = \angle PQR$

Also $\quad \angle 2 = \angle PRQ$

And $\quad \angle SPT = \angle QPR$

(common)

$\Delta PTS \sim \Delta PRQ$

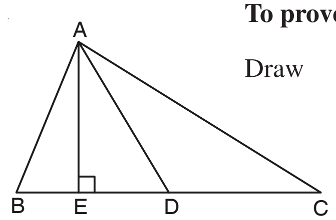
(By AAA similarity criterion)

[CBSE Marking Scheme, 2015] [1 + 1 + 1]

OR

To prove : $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}(BC)^2$

Draw $\quad AE \perp BC$



[1]

In ΔABE, $AB^2 = AE^2 + BE^2$ (Pythagoras theorem)

or $\quad AB^2 = AD^2 - DE^2 + (BD - DE)^2$

$= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$

$\therefore \quad AB^2 = AD^2 + BD^2 - 2BD \times DE$ ...(*i*)

In ΔAEC, $\quad AC^2 = AE^2 + EC^2$ **[1]**

or $\quad AC^2 = (AD^2 - ED^2) + (ED + DC)^2$

$= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC$

or $\quad AC^2 = AD^2 + CD^2 + 2ED \times CD$

or $\quad AC^2 = AD^2 + DC^2 + 2DC \times DE$ ...(*ii*)

Adding eqns. (*i*) and (*ii*),

$AB^2 + AC^2 = 2(AD^2 + BD^2) \quad (\because BD = DC)$

$= \left[2AD^2 + 2\left(\frac{1}{2}BC\right)^2\right] = 2AD^2 + \frac{1}{2}BC^2$ (as BD = BC) **[1]**

Hence Proved.

31. LHS $= \dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$

$$= \frac{(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)}$$

$$= \frac{1+1}{\sin^2\theta - 1 + \sin^2\theta} = \frac{2}{2\sin^2\theta - 1} = \text{RHS}$$

[1 + 1 + 1][CBSE Marking Scheme, 2015]

OR

To prove: $\dfrac{1}{\text{cosec A} - \cot A} - \dfrac{1}{\sin A} = \dfrac{1}{\sin A} - \dfrac{1}{\text{cosec A} + \cot A}$.

$$\frac{1}{\text{cosec A} - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec A} + \cot A}$$ **[1]**

or $$\frac{1}{\text{cosec A} - \cot A} + \frac{1}{\text{cosec A} + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$$

or $$\frac{1}{\text{cosec A} - \cot A} + \frac{1}{\text{cosec A} + \cot A} = \frac{2}{\sin A}$$ **[1]**

L.H.S. $= \dfrac{1}{\text{cosec A} - \cot A} + \dfrac{1}{\text{cosec A} + \cot A}$ or $\dfrac{\text{cosec A} + \cot A + \text{cosec A} - \cot A}{(\text{cosec A} - \cot A)(\text{cosec A} + \cot a)}$

or $\dfrac{2\text{cosec A}}{\text{cosec}^2 A - \cot^2 A}$ or $\dfrac{\frac{2}{\sin A}}{1} = \dfrac{2}{\sin A}$ = R.H.S. **[1]**

Hence Proved

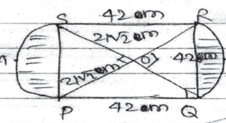
32.

	PR^2	$= 42^2 + 42^2$
		$= 2 \times 42^2$
∴	PR	$= 42\sqrt{2}$
∴	OP	$= 21\sqrt{2}$ **[1]**
	Area of one flower bed	= Area of segment of circle with centre angle 90°
		$= \frac{22}{7} \times 21\sqrt{2} \times 21\sqrt{2} \times \frac{90}{360} - \frac{1}{2} \times 21\sqrt{2} \times 21\sqrt{2}$ **[1]**
		$= 693 - 441 = 252 \text{ m}^2$
∴	Area of two flower beds	$= 2 \times 252 = 504 \text{ m}^2$ **[1] [CBSE Marking Scheme, 2017]**

CBSE Topper's Solution, 2015

Given, PQRS is a square lawn of side, PQ = 42 cm

Here, Two circular flower beds are drawn on sides PS and QR with O as centre.



Diagonals of sq. bisect each other at O.

Now, In right ΔRQP, By pythagoras theorem

$PR = \sqrt{(RQ)^2 + (PQ)^2} = \sqrt{(42)^2 + (42)^2}$

$= \sqrt{2(42)^2}$

$= 42\sqrt{2}$ cm

And, as we know that diagonals of a square bisect each other at 90°

$\therefore OS = OP = \frac{42\sqrt{2}}{2} = 21\sqrt{2}$ cm

And, $\angle POS = \angle ROQ = 90°$

Now, Req. shaded area = Area of 2 segments with $r = 21\sqrt{2}$ cm & $\theta = 90°$

$= 2 \times \frac{r^2}{2}\left[\frac{\pi\theta}{180} - \sin\theta\right]$

$= r^2\left[\frac{\pi \times 90°}{180} - \sin 90°\right]$

$= (21\sqrt{2})^2\left[\frac{22}{14} - 1\right]$

$= 441 \times 2\left[\frac{22-14}{14}\right] = 441 \times 2\left[\frac{8}{14}\right]$

$= 63 \times 8$

$= 504$ m²

33. Largest possible diameter of hemisphere = 10 cm

$\therefore$ radius = 5 cm

Total surface area = $6(10)^2 + 3.14 \times (5)^2$

Cost of painting = $\frac{678.5 \times 5}{100} = \frac{₹\ 3392.50}{100}$ = ₹ 33.9250 = ₹ 33.93

[3] [CBSE Marking Scheme, 2015]

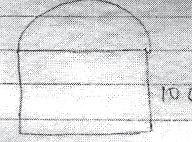
CBSE Topper's Solution, 2015

Here, side of a cubical block = a = 10 cm

A hemisphere surmounts the cube.

$\therefore$ Diameter (largest) of hemisphere = 10 cm

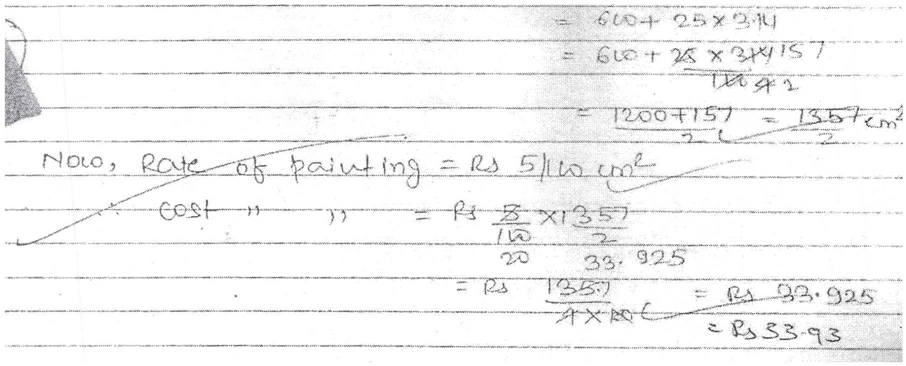
$\Rightarrow$ radius (r) = 5 cm



Now, T.S.A of solid = (T.S.A of cube − area of base of hemisphere) + C.S.A of hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2$

$= 6a^2 + \pi r^2 = 6(10)^2 + \pi(5)^2$



$= 600 + 25 \times 3.14$

$= 600 + \frac{25 \times 3.14}{100}$

$= \frac{1200+157}{2} = \frac{1357}{2} \text{ cm}^2$

Now, Rate of painting = Rs 5/100 cm²

∴ cost " " = Rs $\frac{5}{100} \times \frac{1357}{2}$

= Rs $\frac{1357}{4 \times 20}$ = Rs 33.925

= Rs 33.93

OR

Volume of cuboid = $4.4 \times 2.6 \times 1 \text{ m}^3$

Inner and outer radii of cylindrical pipe = 30 cm, 35 cm

∴ Volume of material used $= \frac{\pi}{100^2}(35^2 - 30^2) \times h \text{ m}^3 = \frac{\pi}{100^2} \times 65 \times 5\, h$

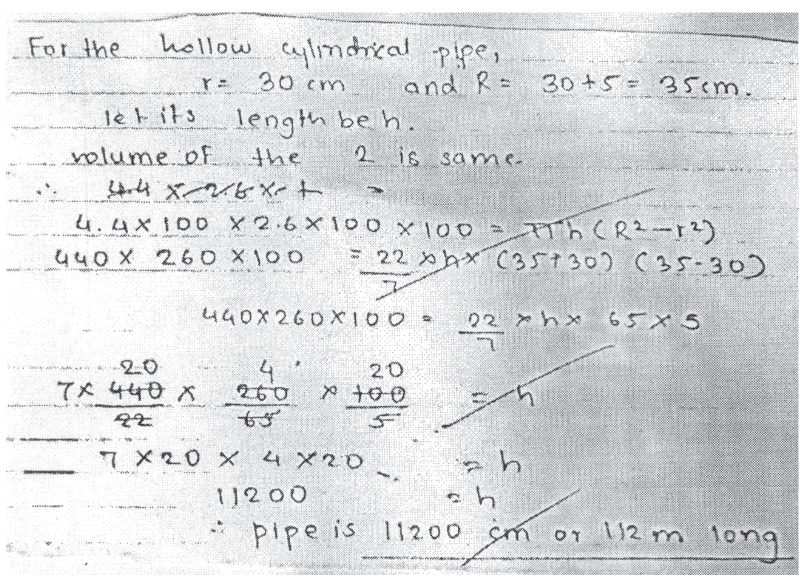
Now $\frac{\pi}{100^2} \times 65 \times 5\, h = 4.4 \times 2.6$

$\Rightarrow \quad h = \frac{7 \times 4.4 \times 2.6 \times 100 \times 100}{22 \times 65 \times 5}$

$\Rightarrow \quad h = 112 \text{ m}$

[3] [CBSE Marking Scheme, 2017]

CBSE Topper's Solution, 2017



For the hollow cylindrical pipe,

r = 30 cm and R = 30+5 = 35 cm.

let its length be h.

volume of the 2 is same.

∴ $4.4 \times 100 \times 2.6 \times 100 \times 100 = \pi h(R^2 - r^2)$

$440 \times 260 \times 100 = \frac{22}{7} \times h \times (35+30)(35-30)$

$440 \times 260 \times 100 = \frac{22}{7} \times h \times 65 \times 5$

$7 \times \frac{440}{22} \times \frac{260}{65} \times \frac{100}{5} = h$

$7 \times 20 \times 4 \times 20 = h$

$11200 = h$

∴ pipe is 11200 cm or 112 m long

34.

C.I.	x	f	$u_i = \frac{x - f}{h}$	$f_i u_i$
0 – 10	5	5	–3	–15
10 – 20	15	x	–2	$-2x$
20 – 30	25	10	–1	–10
30 – 40	35	12	0	0
40 – 50	45	7	1	7
50 – 60	55	8	2	16
Total		**42 + x**		**$-2x - 2$**

[2]

A = Assumed mean = mid point of class 30 – 40 = 35

$$\text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\Rightarrow \quad 31.4 = 35 + \frac{-2x-2}{42+x} \times 10$$

$$\Rightarrow \quad (2x + 2)10 = (42 + x)(3.6)$$

$$\Rightarrow \quad 20x + 20 = 151.2 + 3.6x$$

$$16.4x = 131.2$$

$$x = 8$$ **[1]**

Section 'D'

35. Suppose B alone finish the work in x days and A alone takes $(x - 6)$ days.

B's one day work $= \frac{1}{x}$ **[1]**

A's one day work $= \frac{1}{x-6}$ **[1]**

and (A + B)'s one day work $= \frac{1}{4}$

According to the question, $\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$

or $x^2 - 14x + 24 = 0$ **[1]**

or $x^2 - 12x - 2x + 24 = 0$

or $x(x - 12) - 2(x - 12) = 0$

or $(x - 12)(x - 2) = 0$

or $x = 12$ or $x = 2$

But x cannot be less than 6. So $x = 12$ **[1]**

Hence, B can finish the work in 12 days.

CBSE Topper's Solution, 2017

Let B complete a work in x days.
Then A takes $x-6$ days to complete it.
Together they complete it in 4 days.
According to work done per day,

$$\frac{1}{x-6} + \frac{1}{x} = \frac{1}{4}$$

$$\therefore \frac{x + x - 6}{x(x-6)} = \frac{1}{4}$$

$$4(2x-6) = x(x-6)$$

$$8x - 24 = x^2 - 6x$$

$$\therefore x^2 - 14x + 24 = 0$$

$$x^2 - 12x - 2x + 24 = 0$$

$$x(x-12) - 2(x-12) = 0$$

$$(x-2)(x-12) = 0$$

$$\therefore x = 2 \text{ or } 12.$$

$x = 2$ is not possible because then $x-6$ is (-4)

$\therefore x = 12.$

So, B takes 12 days to finish the work.

OR

Since (–5) is a root of given quadratic equation

$$2x^2 + px - 15 = 0$$

and the quadratic equation

$$p(x^2 + x) + k = 0$$

has equal roots,

then $2(-5)^2 + p(-5) - 15 = 0$ **[1]**

$$50 - 5p - 15 = 0$$

$$5p = 35$$

$\Rightarrow$ $p = 7$ **[1]**

Now, $p(x^2 + x) + k = 0$

has equal roots

$px^2 + px + k = 0$ **[1]**

$$b^2 - 4ac = 0$$

$$(7)^2 - 47 \times k = 0$$

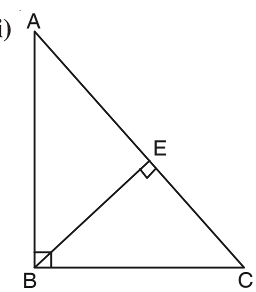
$$28k = 49$$

$$k = \frac{49}{28} = \frac{7}{4}$$

Hence, $p = 7$ and $k = \frac{7}{4}$

[1]

35. (i)



Given : AB ⊥ BC

Construction : Draw BE ⊥ AC

To Prove : $AB^2 + BC^2 = AC^2$

Proof : In ΔAEB and ΔABC ∠A = ∠A (Common)

∠E = ∠B (each 90°)

ΔAEB ~ ΔABC (By AA similarity)

or $\frac{AE}{AB} = \frac{AB}{AC}$

or $AB^2 = AE \times AC$ ...(*i*)

Now, in ΔCEB and ΔCBA, ∠C = ∠C (Common)

∠E = ∠B (each 90°)

$$\Delta CEB \sim \Delta CBA$$

(By AA similarity)

or $$\frac{CE}{BC} = \frac{BC}{AC}$$

or $$BC^2 = CE \times AC \quad ...(ii)$$ **[1]**

On adding eqns. (*i*) and (*ii*),

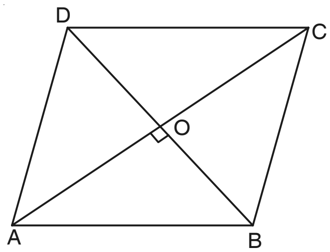
$$AB^2 + BC^2 = AE \times AC + CE \times AC$$

or $$AB^2 + BC^2 = AC(AE + CE)$$

or $$AB^2 + BC^2 = AC \times AC$$

$\therefore$ $$AB^2 + BC^2 = AC^2$$ **[1]**

(ii)



Given: ABCD is a rhombus,

Construction: Draw diagonals AC and BD

$\therefore$ $$AO = OC = \frac{1}{2}AC$$

and $$BO = OD = \frac{1}{2}BD$$

$$AC \perp BD$$

To Prove : $4AB^2 = AC^2 + BD^2$

Proof : $\angle AOB = 90°$

(Diagonals of rhombus bisect each other at right angle)

$$AB^2 = OA^2 + OB^2$$

or $$AB^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$ **[1]**

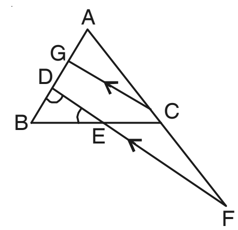
$$AB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$$

$$4AB^2 = AC^2 + BD^2$$ **[1]**

Hence proved.

OR

Construction: Draw CG || FD



Given : $\angle BED = \angle BDE$

or $$BE = BD = EC \quad ...(i)$$

(Given that E is mid-point of BC)

In ΔBCG, DE || GC

or $$\frac{BD}{DG} = \frac{BE}{EC} = 1 \text{ (from } (i))$$ **[1]**

or $$BD = DG = EC = BE$$

[using (i)]

In ΔADF CG || FD

or $$\frac{AG}{GD} = \frac{AC}{CF}$$ (By BPT) **[1]**

adding 1 on both sides,

$$\frac{AG}{GD} + 1 = \frac{AC}{CF} + 1$$ **[1]**

or $$\frac{AD}{GD} = \frac{AF}{CF}$$

$\therefore$ $$\frac{AF}{CF} = \frac{AD}{BE}$$ [Using (*i*)] **[1]**

Hence proved.

37. Step of Construction :

(i) Construct a ΔABC in which BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$.

(ii) Draw a ray BX such that $\angle CBX$ acute angle.

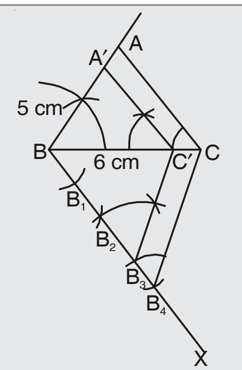
(iii) Locate 4 points B_1, B_2, B_3 and B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(iv) Join B_4C.

(v) Through B_3 draw a line parallel to B_4C which meet BC at C′.

(vi) Through C draw a line parallel to AC which meet AB at A′.

(vii) $\Delta A'BC'$ is the required triangle. **[2]**



[2]

([CBSE Marking Scheme, 2018]

38. **(i)** Let AG be the height of the balloon from the ground and C, D be the positions of the windows, such that BC = 1.5 m, CD = 3 cm.

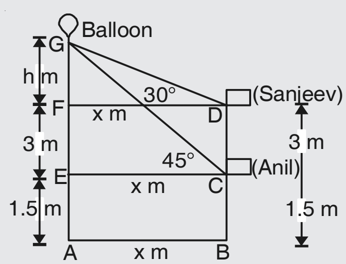
At points C and D, angles of elevation of balloon are 45° and 30°. **[½]**

Draw perpendicular lines EC and FD on AG.

Then, $\angle ECG = 45°$, $\angle FDG = 30°$

$BC = AE = 1.5$ m

and $CD = EF = 3$m



Let AB = CE = DF = x m and FG = h m

In right angled ΔECG,

$$\tan 45° = \frac{EG}{EC} = \frac{3+h}{x}$$

$$\Rightarrow \quad 1 = \frac{3+h}{x}$$

$$\Rightarrow \quad x = 3 + h$$

[∵ tan 45° = 1] ...(*i*) **[½]**

In right angled ΔFDG,

$$\tan 30° = \frac{GF}{DF} = \frac{h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h$$

$$\left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$ **[½]**

On putting $x = \sqrt{3}h$ in Eq. (*i*), we get

$$3 + h = \sqrt{3}h \Rightarrow h(\sqrt{3}-1) = 3$$

$$\Rightarrow h = \frac{3}{(\sqrt{3}-1)} \Rightarrow h = \frac{3}{1.732-1} = \frac{3}{0.732}$$

$$\therefore \quad h = 4.098 \text{ m}$$

Hence, the height of balloon from the ground = 4.098 + 3 + 1.5 = 8.598 m = 8.6 m. **[½]**

(ii) The person who makes small angle of elevation is more closer to the balloon. Hence, Sanjeev is more closer to the balloon. **[½]**

(iii) No, when the balloon is moving towards the building, then the angle of elevation will automatically increase. **[½]**

(iv) Windows are the most important part of any building. They add different values to it, like they are useful for the proper ventilation, which is very much required as natural air keeps the building fresh and suffocation free. **[1]**

[CBSE Marking Scheme, 2015]

39. Given, Height of cylinder = 15 cm

Its diameter = 12 cm

radius = 6 cm

radius of cone = 3 cm

and height of cone = 9 cm **[1]**

Let the number of toys recast be n.

∴ Volume of n conical toys = Volume of cylinder **[1]**

$$\Rightarrow n \times \frac{1}{3}\pi \times 3 \times 3 \times 9 = \pi \times 6 \times 6 \times 15$$

$$\Rightarrow \quad n = \frac{6 \times 6 \times 15}{3 \times 9}$$ **[1]**

$$\Rightarrow \quad n = 20$$ **[1]**

Hence, the number of toys = 20

40.

Students	*c.f.*
Less that 7	20
Less than 9	38
Less than 11	60
Less than 13	85
Less than 15	105
Less than 17	120
Less than 19	130

[1]

This curve is the required cumulative frequency curve or an ogive of the less than type.

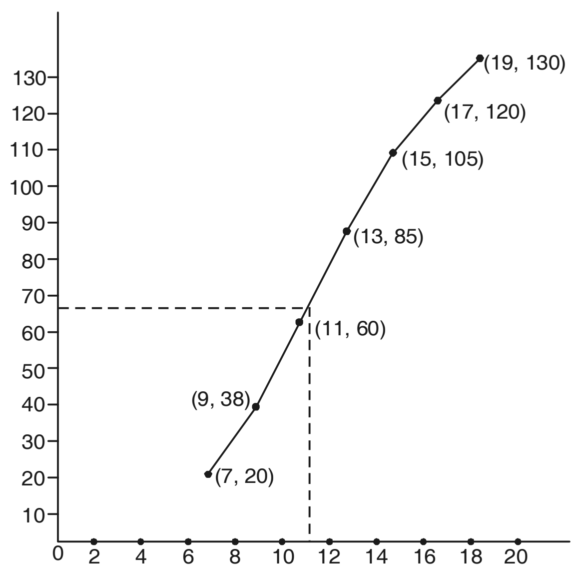
Here, $N = 130$

So, $\frac{N}{2} = \frac{130}{2} = 65$ **[1]**

Now, we locate the point on the ogive whose ordinate is 65. The x-co-ordinate corresponding to this ordinate is 11.4

Hence, the required median on the graph is 11.4. **[1]**

Units on: x-axis 1 cm = 2
x-axis 1 cm = 10



[1]

OR

Modal class = 11 – 13

$l = 11, f_1 = 95, f_0 = 41, f_2 = 36, h = 2$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times 2$$

$$= 11 + \frac{95 - 41}{190 - 41 - 36} \times 2$$

$$= 11 + \frac{54}{113} \times 2$$ **[½]**

Mode = 11 + 0.95 = 11.95 **[½]**

Age	x_i	f_i	$f_i x_i$
5 – 7	6	67	402
7 – 9	8	33	264
9 – 11	10	41	410
11 – 13	12	95	1140
13 – 15	14	36	504
15 – 17	16	13	208
17 – 19	18	15	270
		$\Sigma f_i = 300$	$\Sigma f_i x_i = 3{,}198$

[2]

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{3{,}198}{300} = 10.66$$ **[1]**